Joint Optimisation of Generation and Storage in the Presence of Wind

by

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Declarations

The work submitted in this thesis is the result of my own investigation, except where stated. It has not been submitted or accepted for any other degree at another university.

Part of the thesis has already been published by the IET Renewable Power Generation under the title - Joint Optimisation of Generation and Storage in the Presence of Wind. It contains material from Section 2.1,3.2, 3.3, 5.4 and Chapter 4.

Abstract

As future grids are becoming more decentralised, I study a stand-alone grid where the penetration of wind energy is high, and exploit a joint planning of energy storage and renewable energy source, as this can potentially result in a more economical and efficient energy system. More specifically, I consider an energy system that consists of a gas-fired plant and a small wind farm with a capacity for energy storage. I assume that the gas-fired plant has a maximum generation that is no more than the electricity consumption. I first propose an optimisation model with known wind speed and electricity demand. Then I gradually extend this deterministic model to study the stochastic nature of the wind speed and electricity demand forecasting. Numerical applications in two chosen locations with different characteristics have been provided for demonstration. In the model extension, I compare battery storage with the other storage technologies by modifying the part of the cost functional, charging/discharging capacity and efficiency rate corresponding to the storage. The optimal solution has changed due to different efficiencies, costs and charging/discharging capacities. Compressed air energy storage and pumped hydroelectric storage may have the advantage in cost, but if a big surplus of energy is needed to get charged within a short time period, batteries might be a better choice as flywheels are very expensive. Furthermore, I include carbon emission modelling from the gas-fired plant by applying a carbon tax and a carbon emission cap. In my system, for a carbon tax to have a similar effect in reducing emissions in comparison to a carbon emission cap, it would need to be very high. Finally, I consider the possibility of connecting my system to the National Grid where I import from, or export to, when my system has an electricity shortage or surplus in meeting the demand. The results provide helpful insights in planning a joint deployment of generation capacity and energy storage and show that the system operates more efficiently and economically when it is connected to the National Grid.

Contents

Chapter 1

Introduction

Currently, electricity is mainly generated from finite resources such as coal and gas, and these resources are declining. In addition, the utilisation of fossil fuels is harmful to the environment as they emit greenhouse gases when burned. In fact, electricity generated from fossil fuels is one of the largest sources of greenhouse gas emissions (EPA, 2011). The release of greenhouse gas into the atmosphere contributes to both climate change and the production of acid rain. Concerns over climate change have led to increased activity to search for alternative energy resources with lower carbon emissions and renewable energy.

In response to the increase in electricity demand and the decline in fossil fuel resources, along with the threat of global warming, it has driven the US government to set an emission target of cutting pollution from these power plants by 30% by 2030. Such aggressive targets to integrate renewable energy have been announced in different forms all over the world. Under the current EU commitment to reduce greenhouse gas emissions to 80 - 95% below 1990 levels by 2050, the UK would need to adopt between 23 - 42% renewable power in order to achieve an 80% reduction in carbon emissions by 2050 (Rhodes, 2010; Abdel-Aal et al., 2014).

Renewable resources such as wind, solar or hydroelectric power do not diminish with their utilisation. There has been a growth in the level of research being conducted in the development of technologies to utilise these resources to obtain the maximum output of electricity. Among the current renewable options, wind power is often considered to be the best due to its cost-effectiveness and the advances in technology (Benitez et al., 2007). With wind being the world's fastest growing energy resource, it has also raised the importance of efficiently managing wind energy and understanding the factors that affect the costs of using wind power. However, the rapid growth in wind power creates many new challenges for the existing power grid. One is the conflict between the high penetration rate and the stochastic availability of wind power. The world needs more wind power integrated into the power grid to meet the increasing need for renewable generation. But increasing the penetration of wind power is a huge challenge for the further development of wind power (Wang et al., 2013).

Renewable energy sources (i.e. wind, solar) are non-dispatchable sources that are both variable and uncertain in nature. One option to deal with the intermittent nature of renewable energy resources is demand response, which allows the consumer to participate in the operation of the electric grid by reducing or shifting their electricity usage during peak periods. Another option to manage the variability and mitigate the uncertainties in renewable energy is by coupling energy storage with renewable generation. Under this option, wind generation from periods with a low price can be shifted to periods with a higher price. Moreover, energy storage devices facilitate power balancing as they delink the times of generation and consumption. They also improve power quality and reliability, defer and eliminate costly upgrades in the transmission network, and can further increase the availability and value of distributed renewable generation. The main barrier to their deployment is the high cost. Owing to their benefits, governments and industries have been investing significantly in the research and development of energy storage technologies.

In this thesis, I study a stand-alone grid where the penetration of renewable energy is high and exploit a joint planning of energy storage and renewable energy source, as it can potentially result in a more economical and efficient energy system. Moreover, the availability of renewable energy source and the electricity consumption level vary significantly in different geographical regions and at different times of the year. This makes the joint capacity optimisation for generation and storage very important when planning such power systems. An optimisation model will be developed, along with numerical applications to demonstrate how the designed model can be applied in practice. Once a model framework is developed, three extensions will be added to increase its flexibility. Among the various types of renewable energy resources, I focus on wind energy, as the UK is one of the best locations for wind energy. For electricity storage, I consider battery storage primarily, but other types of storage technologies will be introduced too. In the next section, I provide a detailed description and discussion of some of the storage technologies considered in this work.

1.1 Electricity storage technology

There is a number of suggested methods for the categorisation of the electrical energy storage technologies; for instance, in terms of their storage duration and response times (IEC, 2011; Luo et al., 2015). One of the widely used methods is based on the form of energy stored in the system as shown in Figure 1.1, which can be categorised into electrical (supercapacitors), magnetic (superconducting magnetic energy storage), mechanical (pumped hydroelectric storage, compressed air storage and flywheels) and chemical (batteries and hydrogen/fuel cells).

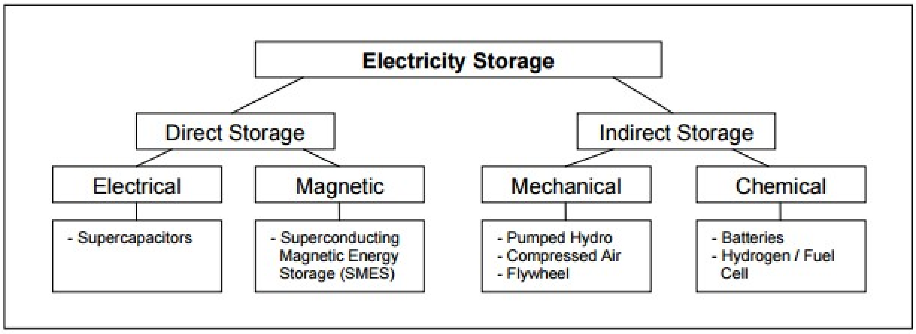

Figure 1.1: Overview of electricity storage technology

For those that belong to the direct storage, electricity is stored in the magnetic or electrical field of a capacitor, whereas for indirect storage technology, electricity is stored as mechanical or chemical energy and then converted back into electricity when needed (Mariyappan et al., 2004). Due to the high cost and inherent technology limitations for direct storage technologies, indirect storage technologies are most widely used. Pumped hydro and battery systems are the most mature technologies. However, in recent years, compressed air storage has gained more attention in research and development because of its potential to store a large amount of electricity.

The characteristics for energy storage vary with its type. For the different types of storage, they have different round-trip efficiency, energy loss over time, maximum capacity and investment cost. For instance, chemical batteries have a relatively high energy efficiency and low energy loss over time, but the maintenance cost is high. Flywheels have a high energy efficiency and a charge/discharge rate but the rate of energy loss over time is high. For hydrogen energy storage, energy loss over time is small, but the energy efficiency is low.

1.1.1 Battery energy storage

The most widely used electrical energy storage technologies in our daily life and industry are rechargeable batteries. Batteries can be used for a variety of applications, for instance, power quality, energy management, ride-through power and transportation systems. Battery energy storage (BES) systems can be constructed within a relatively short length of time - under 12 months (Smith et al., 2008; Chen et al., 2009; Luo et al., 2015). The location for installation of batteries is quite flexible; it can either be inside of a building or close to the facilities where required. However, the relatively low cycling times and high costs are considered to be the main barriers when implementing such facilities.

1.1.1.1 Lead-acid batteries

Lead-acid batteries are the most widely used rechargeable batteries (Ibrahim et al., 2008; Chen et al., 2009). The typical applications include stand-alone systems with photovoltaic, battery systems for mitigating the output fluctuations from wind, emergency power supply systems, and as starter batteries in vehicles (IEC, 2011).

Electric batteries store energy in electrochemical form. During discharging, the negative electrode releases electrons to flow through the electric load that is connected to the battery to release energy. Electrons are then transported to the positive electrode for electrochemical reduction. The process is reversed for charging.

Lead acid batteries have a relatively high cycle efficiency (up to 90%) and fast response time (Kondoh et al., 2000; Chen et al., 2009; Hadjipaschalis et al., 2009; Beaudin et al., 2010). However, their main limitations in installations include relatively low cycling time (up to $\sim$ 2000), energy density (50 - 90 Wh/L) and specific energy (25 - 50 Wh/kg) (Farret and Simões, 2006; Baker, 2008; Chen et al., 2009). Meanwhile, they perform poorly at low temperature which increases the cost (Luo et al., 2015).

Currently, the research and development of lead acid batteries concentrate on the innovation of materials for the improvement of performance (i.e. increased cycling times and deep discharge capability) and implementations for applications in wind, PV integration and automotive sectors (Luo et al., 2015). There are advanced lead acid batteries with fast response times, comparable to flywheels being developed, such as the Xtreme Power advanced lead acid dry cell (Rastler, 2010).

1.1.1.2 Lithium-ion batteries

Lithium-ion (Li-ion) batteries can be an ideal choice where the response time (milliseconds), small dimension and weight of equipment ($\sim$ 1500 - 10,000 W/L, $\sim$ 75 - 200 Wh/kg, $\sim$ 150 - 2000 W/kg) are considered to be important (Chen et al., 2009; Hadjipaschalis et al., 2009; IEC, 2011). Li-ion batteries also have a high cycle efficiency (up to $\sim$ 97%), but the depth of discharge has a big impact on the battery life (Chen et al., 2009; IEC, 2011; Luo et al., 2015).

Recent research activities have been focusing on how to increase battery power capability with the use of nanoscale materials and enhancing battery specific energy by developing advanced electrode materials and electrolyte solutions (Luo et al., 2015). Quite a few companies have been using Li-ion batteries in the utility-scale energy market. For example, the largest European Li-ion battery trial is currently in process in the UK. This project aims to install a Li-ion battery 6 MW/10 MWh at a primary substation in Bedfordshire to assess the cost-effectiveness of the energy storage as part of the UK Carbon Plan (Tweed, 2013; Luo et al., 2015). It has been claimed that this storage can be used for balancing the intermittency of the wind and other renewable energy resources, and it can help to save more than $9 million compared to traditional system upgrades (Tweed, 2013). At the end of 2013, Toshiba made an announcement to install a 40 MW/20 MWh Li-ion battery in Tohoku to help the integration of renewable energy into the grid (Daly, 2014).

1.1.2 Compressed air energy storage

Compressed air energy storage (CAES) is another relatively mature technology, which has been used since the 19th century for different industrial applications. The first CAES facility, the Huntorf plant in Germany, began to operate in 1978 (Succar and Williams, 2008; Raju and Khaitan, 2012; Carnegie et al., 2013; Madlener and Latz, 2013). It was used to store off-peak base load energy from a nuclear power plant. Recently, the facility has been used as a spinning reserve for industrial customers and to level variable power from integrated wind energy (Carnegie et al., 2013). Another commercial CAES plant located in McIntosh, US, started to operate in 1991 (Succar and Williams, 2008; Raju and Khaitan, 2012; Carnegie et al., 2013; Madlener and Latz, 2013). It is now used for load management, peaking power, ramping duty, synchronous condenser duty, and spinning reserve applications (Carnegie et al., 2013). These two CAES plants have consistently shown good performance with 91.2 - 99.5% running reliabilities (Succar and Williams, 2008; Frank and Levine, 2011).

CAES involves compressing and storing air to store energy using geological underground voids or vessels. When needed, the released air is heated and expanded to drive the gas turbine to generate electricity. The compressor and expander in a CAES system are different from a conventional combustion turbine in the way that they are separate and operate independently despite being mounted on a single shaft in a combustion turbine. The compressor in a combustion turbine is driven by some of the power generated in the expander. A number of storage vessels can be used; for example, pipes, hard rock caverns, underwater bladders and above-ground tanks. Meanwhile, a variety of fuels, such as natural gas, hydrogen can be used in the combustion process.

There are three major CAES technologies, diabatic, adiabatic, and near-isothermal. Diabatic uses the heat added in the expansion process to increase the power capacity, it is the most technologically developed form of CAES and is used in the Huntorf and McIntosh installations (Carnegie et al., 2013). Adiabatic CAES uses a thermal storage device to capture heat expelled in the compression process and reheat the air using the stored thermal energy in the expansion process. Near-isothermal CAES technology compresses and expands slowly to maintain constant air temperature, and eliminates the need for burning fossil fuels to reheat the air during expansion which increases efficiency.

CAES systems can be built in small and large capacities, they provide moderate response time and a good partial-load performance (Luo et al., 2015). Large scale CAES plants are often used in grid applications such as peak shaving, load shifting and frequency control. CAES can also be coupled with intermittent renewable energy applications to smooth the power output, and have gained extensive attention from academic researchers and industrial sectors (RWE, 2010; Succar et al., 2012; Madlener and Latz, 2013).

CAES systems can provide a power output of over 100 MW with a single unit, a rapid start-up time, and has a much longer lifetime than batteries. However, major barriers to implement such systems are the identification of suitable geographical features and the relative low round-trip efficiency (about 70%).

1.1.3 Flywheel energy storage

Flywheel energy storage (FES) systems are usually categorised into low-speed and high-speed. High-speed systems are made of high strength and low-density composite materials and are more compact than low-speed metallic wheels. Low-speed FES systems are mainly used for short-term and medium/high power applications, whereas, high-speed FES systems are mainly used for high power quality and ride-through power service in traction and the aerospace industry (Díaz-González et al., 2013; Luo et al., 2015). The first generation of flywheels has been available from 1970. Since then, they have been used for maintaining power reliability and quality by regulating frequency and providing protection in power supplies against transient interruptions (Carnegie et al., 2013).

The main components of a flywheel are the rotating body in a compartment, the transmission and the bearings. A flywheel makes use of the mechanical inertia contained within the rotating flywheel to store energy. To retrieve the stored energy, the process is reversed with the motor that accelerated the flywheel to act as a brake to extract energy from the rotating flywheel. The amount of stored energy depends on the mass, the speed and the configuration of the flywheel.

Flywheels are more suitable for improving power quality by smoothing fluctuations in generation due to the ability of rapid charge and discharge. They have excellent cycle stability, long lifetime, high power density and good efficiency rate. Another advantage is that they require little maintenance and use environmentally inert material. The drawbacks are the high cost, wide speed variation when extracting energy and high levels of self-discharge.

1.1.4 Pumped hydroelectric storage

Pumped hydroelectric storage (PHS) has a long history, a large energy capacity and high technical maturity. With an installed capacity of 127 GW in 2010 (Figure 1.2), PHS represents more than 99% of the world's bulk storage capacity excluding fossil fuels and contributes 3% of the global generation (IEA, 2014; Luo et al., 2015). PHS plants were first established in Italy and Switzerland in the 1890s. Their main applications are energy management via time-shift and supply reserve (IEC, 2011). The first PHS plant in the US began its operation in 1929 and supported balance generation with load. Since then, PHS has also been used for time-shifting, smoothing, and firming of intermittent renewable generation (Carnegie et al., 2013).

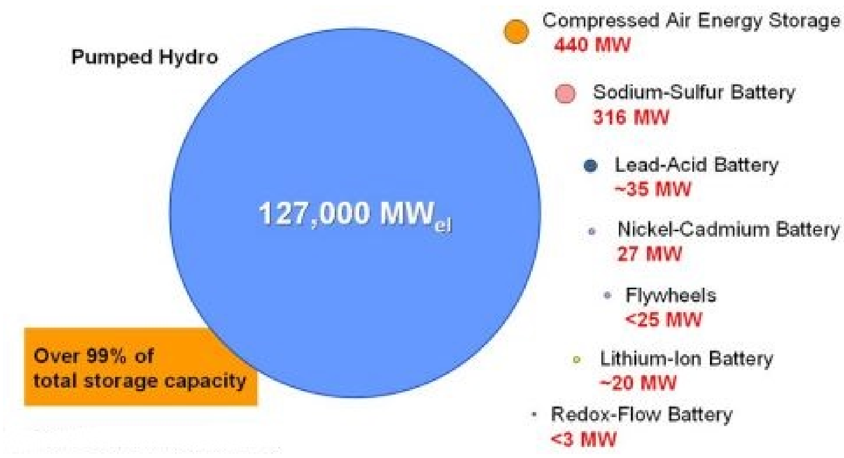

Figure 1.2: Global installed grid-connected electricity storage capacity (MW) by 2010

A typical PHS plant makes use with the differential in heights between the two reservoirs to store energy. A body of water at a high elevation represents stored (potential) energy. Electricity is produced in the discharging process by releasing the water from the high reservoir, which then flows through hydroturbines into the lower reservoir. The water is pumped back up to recharge the upper reservoir, which represents charging.

Various PHS plants exist with approximately 70 - 85% efficiency and power ratings ranging 1 - 3000 MW (Arántegu et al., 2012; Carnegie et al., 2013; Luo et al., 2015). PHS plants have a large power and energy capacity, a very long lifetime and a fast response time. However, due to the restriction of site selection, they suffer long construction time.

1.2 Thesis structure

After reviewing the existing literature on renewable generation with energy storage and the various techniques for modelling wind speed and electricity demand in Chapter 2, I then introduce the basic optimisation model in Chapter 3, which computes the optimal capacities for the gas-fired plant, the wind turbine and the storage that make up a decentralised grid with known wind speed and electricity demand.

In Chapter 4, I present the stochastic modelling of wind speed and electricity demand forecasting. For wind speed, the stochasticity is captured by applying a MC model on historical data, whereas for electricity demand, a mathematical model comprised of three parts is applied. The first part of the electricity demand model is to model the yearly changes, then the modelling of weekly residual variations within a year, and followed by the hourly variations within a week. The overall model is constructed by combining all three parts.

In Chapter 5, I compare the effects of the different types of storage technologies. Since different storage technologies have different characteristics (e.g. efficiency rate, lifetime, cost), it would be interesting to see how the optimal solution changes. In the second extension, I include the carbon emission modelling from the gas-fired plant. Finally, I assess the effect of connecting the decentralised system to the National Grid by considering imports and exports of electricity. By allowing the interconnection with the National Grid, the model would be more flexible and the system could operate more efficiently.

In Chapter 6, I provide some concluding remarks and discuss possible future work. There are two appendices attached after the conclusion. Appendix A contains the MC modelling on electricity demand, whereas, Appendix B contains all the numerical examples from the chapters.

Chapter 2

Literature review and research contribution

Wind energy is one potential renewable energy that is currently available. Many countries have used wind power to meet their electricity demand. To mitigate the uncertainty of wind power, it can be combined with electricity storage. In this chapter, I first review the related work on the optimisation in energy systems, then review the different modelling approaches on wind speed and electricity demand in order to select the most appropriate model for my study. Finally, I propose the research objective and highlight the main research contributions.

2.1 Optimisation in energy systems

A significant body of work has been reported by many researchers on the optimisation of energy systems with energy storage and renewable generation. I review some of them here.

There is a stream of work that focuses on joint optimisation; for example, Liu et al. (2016) investigate the joint optimisation of generation and storage with renewable supply (wind). They study a stand-alone grid with connection to the National Grid and the constrained non-linear problem is formulated into a cost minimisation framework and then solved in MATLAB using the nonlinear optimisation solver. Their results provide very helpful insights in planning the joint deployment of generation capacity and energy storage. The stochastic nature of wind speed and electricity demand forecasting have been taken into account. However, they have considered only batteries as energy storage. The loss of load is not considered either. Yang and Nehorai (2014) study the problem of jointly optimising multiple energy storage, renewable generator, and diesel generator capacities in the context of a micro-grid with a small carbon print. The joint optimisation exploits the different characteristics of multiple energy storage types as well as the availability of different sources of renewable energy. Due to the use of large volumes of historical data, and to mitigate the large dimensionality of the optimisation problem, they re-formulate the original optimisation problem into a consensus problem, which is then solved in a parallel distributed manner. Again, the results are useful in making decisions on storage and generation planning in a power grid with a high penetration level of renewable energy. However, the proposed framework only optimises the hybrid energy storage and generation system under the assumption of perfect forecasting for demand and renewable energy generation, hence, the stochastic nature of demand and renewable generation is not taken into account. Furthermore, the case of an interconnected network with the National Grid or other micro-grids is not considered. Garcia-Gonzalez et al. (2008) investigate the combined optimisation of a wind farm and a pumped storage facility using a two-step stochastic optimisation approach, from the perspective of a generation company. The optimisation produces optimal operation strategies of the facilities and optimal bids for the day-ahead spot market. However, the optimal planning of generation and energy storage capacity is not considered.

In the literature on wind-storage systems, Denholm and Sioshansi (2009) and Fertig and Apt (2011) look at the interplay of storage and transmission capacity. Both studies evaluate the value of storage and assume deterministic processes for price and wind energy. The former studies the best location for storage when transmission capacity is binding; the latter examines the optimal sizing of storage and transmission capacity. In the study of (Abbey and Joos, 2009), the authors look at the optimal rating of energy storage in a wind-diesel isolated grid and show that high wind penetration could potentially result in significant cost savings with respect to fuel and operating costs. There are other studies which focus on the value of using energy storage in managing the stochastic nature of the wind within power systems, and the broader economics of energy storage and wind; for example, (Denholm et al., 2005; DeCarolis and Keith, 2006; Abbey and Joos, 2007; Black and Strbac, 2007; Gonzalez et al., 2008). Most studies have paid particular attention to the engineering aspects of wind integration such as, grid stability, load-balance, and system security. Some have shown the benefits of using energy storage as means of managing wind uncertainty and variability.

There are also studies that look at the problems of energy storage in the presence of distributed power generation systems in cases of a balanced and unbalanced electrical grid. Abbassi and Chebbi (2012) has studied and developed supervisory algorithms for the optimal operation of a DC-coupled wind/photovoltaic hybrid system equipped with battery storage, which delivers power to the grid in the presence of a conventional generator. Under the developed supervisory algorithms, the service continuity is assured by meeting the energy demand while optimising the use of conventional sources, except in the case of a major deficit by renewables. Their algorithm has been tested in MATLAB and the results have confirmed the reliability of proposed strategy. However, they claim that this strategy must be tested in real time when many disturbances occur in the electrical network. Meanwhile, Abbassi et al. (2015) investigate the main concerns for the benefits of a grid-connected hybrid system with multiple renewable sources attached with storage. In particular, they have looked into an integrated renewable energy conversion system associated with a storage system under unbalanced voltage conditions. In this work, special attention has been paid to power quality issues and the dynamic performances of the control algorithm.

2.2 Wind speed

Electricity produced from burning of fossil fuels emits gases into the atmosphere which cause global warming. This motivates researchers to search for clean energy such as wind. Wind has shown great potential in delivering clean energy and recent advances in wind turbine technology have led to significant growth of wind power generation across the world. However, the benefits of wind energy are accompanied by several challenges to power system operators; for example, high variability, limited dispatchability and predictability. Therefore, it is necessary to develop new methods that can assist power operators in analysing the impact of the stochastic behaviour of wind on power system operation and planning.

There are several parameters related to wind energy but wind speed is the mostly frequently researched parameter (Sanusi et al., 2013). In my study, I have obtained only the wind speed data. Many models have been used to describe the variational behaviour of wind speed and they can be divided into physical, statistical and hybrid models (Sheela, 2011). The physical models employ meteorological and topological information, statistical models estimate a statistical relationship between the input data and the wind speed generation, and, hybrid models are combinations of both (Sheela, 2011).

These forecasting models can be divided into ultra-short-term, short-term, medium-term and long-term forecasting models. Ultra-short-term is typically from few minutes to 1 hour ahead; short-term is from 1 hour to several hours ahead; medium-term is from several hours to 1 week ahead and long-term can range from 1 week to 1 year or more ahead (Chang, 2014). Ultra-short-term forecasting is important for electricity market clearing, regulation actions and real-time grid operations; short-term is mainly used for economic load dispatch planning, operational security within the electricity market and load reasonable decisions; medium-term is essential when making decisions on unit commitment, reserve requirement and generator online or offline; long-term forecasting applies to operation management, maintenance planning, design of wind farms and the optimal operating cost (Chang, 2014).

There are many well-established models for wind speed modelling, but in this thesis, I review only some of the statistical models (i.e. Weibull, Rayleigh, Markov chain model) and time series models (i.e., ARMA, ARIMA) used for wind speed modelling (Table 2.1) as they are easily implemented in different softwares. Here, ARMA stands for auto-regressive moving average and ARIMA stands for autoregressive integrated moving average. In the next section, I will provide a detailed description and discussion on each of these models and conclude which model is selected for my study.

Statistical model	Time series model
Probabilistic model	ARMA
Markov chain model	ARIMA

Table 2.1: Models used for wind speed modelling

2.2.1 Statistical models

2.2.1.1 Probabilistic models

Since the speed of the wind is continuously changing, this makes it desirable to be described by the probabilistic models. In the literature, several distributions have been proposed for wind speeds such as the Weibull distribution, the Gamma distribution, the Rayleigh distribution, etc. Among the proposed probability density functions, the Weibull distribution is a mathematical idealisation of the distribution for wind speed over time (Odo et al., 2012) and it is also highly recommended in many books related to wind energy.

The probability density function (PDF) of the wind speed described by a Weibull distribution is,

$$f(v) = \begin{cases} 0 & \text{, if } v < 0, \\ \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} \exp\left(-\left(\frac{v}{c}\right)^{k}\right) & \text{, if } v \geq 0. \end{cases}$$

where c and k are known as the scale and shape parameters of the Weibull distribution respectively. The shape parameter k specifies how sharp the peak of the curve is, while the scale parameter c affects how wide the curve stretches. These parameters can be estimated by the method of Maximum Likelihood Estimator.

Islam et al. (2011) use a two-parameter Weibull distribution function for wind speed forecasting and assess wind energy potentiality at Kudat and Labuan, Malaysia. Celik (2003a) uses Weibull representative wind data instead of the measured data in time series format for estimating the wind energy, and shows that the estimated wind energy is highly accurate. Celik (2003b) conducts statistical analysis of wind data at the southern region of Turkey and summarises that the Weibull model is better than the Rayleigh model in fitting the measured data distribution. Akdag et al. (2010) discuss the suitability of a two-parameter Weibull distribution and two-component mixture Weibull distribution (WW-PDF) to estimate wind speed characteristics. Carta and Ramírez (2007) use WW- PDF as it is able to represent heterogeneous wind regimes in which there is evidence of bimodality, bitangentiality or unimodality. The maximum likelihood and least-square methods were used to estimate the parameters of WW- PDF.

In recent years, mixture distributions have been frequently used to describe the wind speed. Jaramillo and Borja (2004) use the mixture Weibull distribution to model bimodal wind speed frequency distribution. Kiss and Jánosi (2008) use Rayleigh, Weibull, and Gamma distributions to model wind speeds over both land and sea. They conclude that generalised Gamma distribution with independent shape parameters for both tails provides an adequate and unified description. Akpinar and Akpinar (2009) use a mixture of truncated Normal distribution and conventional Weibull distribution to model wind speeds. Chang (2011) has employed a combination of Gamma and Weibull distribution and a mixture function of two-component truncated Normal distribution for the wind speed modelling.

Although the Weibull distribution is a convenient and powerful approach, it is based on empirical more than physical justification, and it display limitations such as it does not take into account the time correlation. In my study, the battery storage is intimately linked to the wind generation, which makes it very necessary to capture the time correlation of wind speeds over time. For this reason, I have not selected the Weibull distribution to simulate the wind speed.

2.2.1.2 Markov chain models

A Markov chain (MC) model is often used due to its high accuracy along with the better capability of reproducing statistical properties of wind speed, and this is the approach selected for my study. Moreover, the use of energy storage in the system makes it essential to capture the correlation structure within the time series over time, which again favours the use of the MC. Details of this method will be discussed in Section 4.1.

A MC is a sequence of random variables $\{X_t\}_{t\geq 0}$ such that, for any $t = 0,1,2,\ldots$, the random variable $\{X_t\}$ can take values from a discrete set of states $\{i_0\ldots,i_N\}$ and satisfies the Markov property for conditional probabilities:

$$\begin{aligned}&P(X_{t+1} = i_{t+1} \mid X_t = i_t, X_{t-1} = i_{t-1,\ldots,} X_0 = i_0)\\&= P(X_{t+1} = i_{t+1} \mid X_t = i_t).\end{aligned}$$

A MC represents a system of elements making transitions from one state to another, in other words, this approach requires the observed time series to be divided into different states. Each state contains wind speeds between certain values. For example, state 1 contains wind speeds below 3 m/s, state 2 includes wind speeds between 3 and 6 m/s, etc. In the MC approach, the order of the MC is the number of previous states that can affect the current state. For instance, in a first order MC, the current state is affected only by the previous state. A second or higher order MC is the process by which the current state depends on two or more previous ones. A MC is characterised by its transition matrix, the higher the order the more complicated the matrix becomes. Thus, a higher order MC is not as common as a first or a second order MC.

There is research that focuses on the effectiveness of the first and second order MC models. Sahin and Sen (2001) use a first order MC model for the synthetic wind speed generation. The generated wind speed data is checked against actual wind speed data to evaluate the suitability of the model. It is found that a first order MC model is sufficient to preserve most of the parameter values, but a second order MC model performs better. Shamshad et al. (2005) examine the effectiveness of the first and second order MC for the synthetic generation of wind speed time series. The comparisons are based on the statistical properties (i.e., mean, median, standard deviation, percentiles, Weibull parameters, etc). They also conclude that a second order MC model generates more accurate results. There are other similar studies that use the first and second order of MC for generating wind speed time series, such as the work carried out by Hocaoglu et al. (2008) and Carpinone et al. (2010). When applying MC models, one important step is to determine the MC state size. Hocaoglu et al. (2008) observe that with an increasing the number of states, it has a significant benefit in terms of quality of the generated data. They construct two different MC models, one with 13 wind speed states and one with 26 wind speed states. The generated data from both models are compared against actual data. They conclude that statistical characteristics are satisfactorily reproduced by the generated data, but increasing the dimension of the state of a MC model leads to more accurate results.

There is also a MC model developed for incomplete data; Karatepe and Corscadden (2013) present a novel approach for accurately modelling and ultimately predicting wind speed for selected sites when there is incomplete data. The application of a seasonal simulation for the synthetic generation of wind speed data is achieved using the MC Monte Carlo technique with only one month of data from each season. The limited data is used to produce synthesised data that sufficiently capture one seasonal variation of wind characteristics. This approach is very useful when a full set of data is unavailable.

Another stream of work studies the semi-MC models on the generation of synthetic wind speed series. D'Amico et al. (2012) propose three semi-MC models with the aim of reproducing the statistical properties. They generate synthetic time series of wind speeds using Monte Carlo simulation. Then they use time-lagged autocorrelation to compare the statistical properties of the proposed models with those of real data, and also with a generated synthetic time series by a simple MC. They conclude that their model performs better than a simple MC in reproducing the statistical properties of wind speed data. To decrease the difference between autocorrelation of actual and simulated data, a third or fourth order semi-MC would be required, however, it is too computationally consuming.

The MC model is a well-established statistical model and it is capable of capturing the time correlation of wind speeds, so I have selected this technique for wind speed modelling in my study.

2.2.2 Time series models

2.2.2.1 ARMA and ARIMA models

Another popular type of model in the literature for wind speed modelling is the time series model. ARMA models are mathematical models of persistence in time series and they are effective in predicting the behaviour of a time series from past data. ARMA is a generalisation of auto-regressive (AR) models and moving average (MA) models. AR models and MA models are combined to form the ARMA model. The order of an AR model is the number of time steps that the model goes back to predict future values, whereas, the order of the MA model is the past difference steps that the model goes in order to predict future values. The AR model includes lagged terms of the time series and the MA model includes lagged terms on the residuals. The ARMA model is formed by combing the lagged terms. A lag operator simply means that it operates on an element of a time series to produce the previous element, for example, a lag operator L is defined by $Ly_t = y_{t-1}$, where Ly_t is the value of the time series at time $t-1$.

The ARMA model is a special case of auto-regressive integrated moving average (ARIMA) model. ARIMA models are often applied in cases where data shows evidence of being non-stationary. In simple terms, stationary means that the probability density of the data remains the same when data is shifted in time. The mean and variance should also be constant if data is shifted.

The general mathematical formulation of ARMA is:

$$X_t + a_1 X_{t-1} + a_2 X_{t-2} + \cdots + a_p X_{t-p} = \varepsilon_t + b_1 \varepsilon_{t-1} + \cdots + b_q \varepsilon_{t-q}$$

where p is the autoregressive order and q is the moving average order, $a_1, \dots, a_p$ are the autoregressive parameters, $b_1, \dots, b_q$ are the moving average parameters and $\varepsilon_t, \dots, \varepsilon_{t-q}$ are the random variables.

The order of ARMA models can be determined by several methods, for example, autocorrelation function and partial autocorrelation function. The order selection is based on the model validity criteria, i.e. the Akaike's information criterion, the minimum description length.

Since wind speed has good succession and randomness, it is appropriate to use ARMA models to forecast wind speed. In (Erdem and Shi, 2011), four approaches (component, traditional-linked ARMA, vector autoregression (VAR) and restricted VAR approaches) are developed based on the ARMA method to forecast the wind speed. They conclude that the VAR approach and the traditional-linked ARMA have very similar performance in wind speed. The component approach performs worse than traditional-linked ARMA for wind speed forecasting. In terms of forecasting performance, there is very little difference between the VAR and the restricted VAR approaches. Li et al. (2011) present an ARMA model together with wavelet transformation for wind speed prediction. It is found that the combined model is very effective in improving the accuracy of forecasting. Palomares-Salas et al. (2009) use an ARIMA model for wind speed forecasting including wind speed measurements. The process of model validation and regression analysis is based on real data. It is concluded that the ARIMA model is better for short time intervals compared to back propagation neural network. Firat et al. (2010) propose a new statistical method based on the AR model and carry out an independent component analysis. Based on the obtained results, the proposed method gives higher accuracy in comparison to direct forecasting.

Although ARMA models are widely used, one of their major disadvantages is the difficulty in identifying the non-linear characteristics of the data. For this reason, I have not selected this method when performing wind speed modelling.

2.3 Electricity demand

The number of research studies on electricity demand has increased significantly in an attempt to understand its nature since the first oil shock in the early 1970s (Dilaver, 2012). Forecasting of electricity demand is essential for power planning and operation. According to Ryan and Plourde (2009), there is no right approach to model electricity demand as the modelling strategies differ according to a range of conditions. Several methods have been developed over the years and demand forecasting can be divided into short-term, medium-term and long-term forecasting. Short-term forecasting is from 1 hour up to 1 week ahead, medium-term forecasting is from a week to a year, and long-term forecasting usually ranges longer than a year. Various models have been proposed for short-term, medium-term and long-term forecasting, including the regression model, the time series approach, the intelligence method, etc. In my study, I aim to forecast electricity demand in the long-term but on a hourly basis, therefore, I review some of the approaches that are used for electricity demand forecasting in the short-term, the long-term and both short- and long-term (Table 2.2).

Short-term	Long-term	Short and long-term
Trend method	End-use approach	ARMA, ARIMA
Similar day approach	Input-output approach	ARMAX
	Econometric modelling	Mathematical model

Table 2.2: Models studied for wind speed modelling

2.3.1 Short-term forecasting

Short-term forecasting ranges from 1 hour to 1 week, and it plays a very important role in the operation of a power system's basic operating functions such as energy transactions, unit commitment, security analysis, economic dispatch, fuel scheduling and unit maintenance (Campillo et al., 2012). The trend method and similar day approach are often used in short-term forecasting.

2.3.1.1 The trend method

The trend method expresses the variable to be predicted as a function of time. It is a non-causal method so it does not explain the behaviour of the trend line, but exclusively makes a projection based on historical data. The main advantage of this method is its simplicity and only historical consumption data is required. It is possible to achieve a high level of accuracy for short-term forecasting. Some of the techniques used for this type of forecasting are multiple regression, exponential smoothing, and stochastic time series, etc. The main limitation of the trend method is that it cannot predict changes in the consumption behaviour and it requires a large amount of historical data.

2.3.1.2 The similar day approach

The similar day approach analyses the natural pattern of the electricity load and the forecasting day's weather features to define specific parameters that can be compared to previous days with similar characteristics. This information is used to create a training data bank to feed pattern recognition tools in order to emulate the non-linear relationships between electricity load and factors that influence it. The most common pattern recognition tools used are artificial neural networks (ANN), expert systems, fuzzy logic and support vector machines (SVM). ANN is still the most used method for this approach due to its ability to learn complex and non-linear relationships, the availability of commercial tools for its implementation, and the high level of accuracy that can be obtained from this approach. However, the limitations of ANN are the accuracy required for the training data set, the impact of the ANN architecture design and the training algorithm selection. In order to overcome ANN limitations, SVM has been used lately for improved short-term forecasting. SVM uses a similar approach to that used by ANN, but offer a higher calculated accuracy and shorter training times (Campillo et al., 2012).

Both the trend method and the similar day approach are often used by utility companies, as they can easily obtain information about the users' energy consumption.

2.3.2 Long-term forecasting

For the long-term forecasting, the different approaches in the literature can be categorised into three main groups: i) end-use modelling; ii) input-output modelling; iii) econometric modelling (Ryan and Plourde, 2009).

2.3.2.1 The end-use modelling approach

End-use approach is about identifying the role of each end-use towards the aggregate energy consumption. This approach is based on the principle that electricity demand is derived from users' demand for individual requirements (i.e. lighting, cooling, etc.), hence, they are suitable for predicting demand changes. This demand prediction capability is necessary for long-term forecasting and helpful for the adoption of energy efficiency programs. The amount of historical data required is less, but much more detailed information is needed about the consumers on which the model is based on (Campillo et al., 2012).

2.3.2.2 The input-output approach

This input-output approach was developed by Wassily Leontief in the late 1920s and early 1930s (Dilaver, 2012). It systematically quantifies the interrelationships between the range of sectors in a complex economic system and is based on a fully determined general equilibrium model (Arbex and Perobelli, 2010). The application of this approach on energy demand enables the estimation of the direct energy demand as well as the indirect energy demand via inter-industry transactions (Bhattacharyya and Timilsina, 2009).

Despite the fact that input-output models provide valuable information about the direct and indirect use of energy sources, this approach needs a huge amount of data and very well-described input and output relations, which are often not available. Another weakness of this approach is it assumes a fixed input-output ratio. Economic policy induces changes in the input-output coefficients, hence, this assumption excludes the probability of inter-fuel substitution and substitution of non-energy inputs (Dilaver, 2012). In addition, the time invariant nature of this assumption cannot adequately capture technological progress (Bhattacharyya and Timilsina, 2009; Arbex and Perobelli, 2010). Technological progress is an important driver of energy demand. Therefore, ignoring technological progress may lead to biased outcomes.

2.3.2.3 The econometric modelling approach

The econometric modelling approach is a quantitative approach which aims to analyse relationships between the dependent variables and independent variables using historical data.

It has been widely used for electricity demand modelling because of the availability of historical observations. It can be applied with sufficiently long historical observations on electricity consumption and explanatory variables such as population, income and price. It also has a strong theoretical background consistent with economic theory, consumer and production theory in particular (Dilaver, 2012). When the required historical data is accessible, this seems to be an effective approach. The matter that differentiates the econometric modelling approach from the first two approaches is that it statistically estimates electricity demand relationships, whereas, the end-use and input-output approaches rely on energy surveys and technical studies which are not always available.

2.3.3 Short-term and long-term forecasting

2.3.3.1 Time series models

In my study, I require a method that is capable of both short-term and long-term forecasting, for example, the ARMA model. ARMA models have been extensively applied in the demand forecasting literature (Chow et al., 2005; Weron, 2006; Taylor et al., 2006). Of those which perform short-term demand forecasting, ARMA and autoregressive moving average with exogenous variables (ARMAX) model are the most popular time series techniques (Chow et al., 2005). When choosing between a univariate (ARMA) and a multivariate (ARMAX) time series model, data availability and time horizon can be used to assess which technique is feasible (Hinman and Hickey, 2009). Weron (2006) concludes that MA models are not particularly useful in forecasting electricity demand, but the combination of the moving average process with an autoregressive model (an ARMA process) is very powerful. Taylor et al. (2006) and Hahn et al. (2009) conclude that univariate models are typically used for very short-term electricity demand forecasts.

Amjady (2001) divide the days into four types (Saturday, Sunday to Wednesday, Thursday and Friday, and public holidays) based on the demand data and use a modified ARIMA model to forecast demand taking into account daily seasonality. These models are then separated into models for hot days (average temperature above 23 degrees) and cold days. It is found that the mean absolute percentage errors vary from 1.48% (Sunday to Wednesday, hot) to 1.99% (public holidays, cold). Soares and Souza (2006) propose a stochastic model that employs generalised long memory (by means of Gegenbauer processes) to model the seasonal behaviour of electricity demand. This research points out that the forecasting error is generally high during the summer due to the use of air conditioning, therefore, including temperature as a variable might be able to help resolve this issue. Temperature is excluded from their model because of data unavailability, and the authors advocate the inclusion of temperature where temperature data is available.

While univariate ARMA models are sufficient for short-term demand forecasting, the literature agrees that including exogenous variables like temperature can potentially improve the forecasting performance (Soares and Souza, 2006; Soares and Medeiros, 2008). Darbellay and Slama (2000) have applied both the univariate model using an ARIMA model and the multivariate model using an ARMAX model that incorporates temperature data. Using hourly demand data from the Czech Republic, the authors find that the ARMAX model is superior.

Among those who have modelled electricity demand using either ARMA or ARMAX models, there has been a consensus that electricity demand data suffers from multiple seasonality. According to Hahn et al. (2009), time series demand data contains three seasonal patterns which are daily, weekly and annual. The daily seasonal pattern reflects a peak and off-peak demand pattern, whereas, the weekly pattern reflects the variation in demand on weekdays versus weekends. The specific weekday pattern differs between different regions and seasons (Hippert et al., 2001). Moreover, electricity demand can vary significantly due to the presence of a holiday and other exceptional events.

Many techniques have been adopted to address the complexity of seasonality in the forecasting of electricity demand in the literature. Soares and Medeiros (2008) introduce a dummy variable for each day of the week in addition to holiday dummies. There are 15 different binary variables included to account for weekly and holiday seasonality. Similarly, Cottet and Smith (2003) include 13 dummy variables, one for each day of the week and six to account for public holidays. They point out that the coefficients estimated for the dummy variable are similar for weekdays Monday to Friday with a slightly lower demand on Friday afternoon and Monday morning. Moreover, they find that holiday demand can vary depending on the holiday, but generally resembles the demand profile seen on Sundays. Alternatively, there is another study that uses seasonal differencing to account for stochastic seasonality (Taylor et al., 2006). They include both a 24th difference to account for daily seasonality and a 168th difference to model weekly seasonality.

2.3.3.2 Mathematical models

Annual and other types of seasonality have been addressed in the literature through the application of Fourier decomposition (Cottet and Smith, 2003; Soares and Medeiros, 2008; Filik et al., 2011). Cottet and Smith (2003) use the Fourier decomposition to account for annual seasonality, whereas, Soares and Medeiros (2008) model the annual cycle as a sum of sines and cosines. Similarly, Filik et al. (2011) propose a novel mathematical method to forecast the electricity demand. This model is constructed using 4 years' demand data with an hourly resolution which was obtained from a Turkish electric power company. The model is a combination of three sub-sections, the first section is for yearly demand variations, the second section is about weekly residual variations within a year and the final section is the hourly variations within a week. To reduce the forecasting error, several mathematical functions are applied at each level. Mean absolute percentage error (MAPE) and root mean square error (RMSE) are used to show the effectiveness of the proposed function. They conclude that a multi-resolution framework is well capable for electricity forecasting.

Unlike the other long-term forecasting model, the model in (Filik et al., 2011) is able to produce results with improved accuracy on an hourly basis. Normally, long-term forecasting is performed on yearly basis, whereas, hourly accuracy is used for short-term prediction. However, this model provides hourly accuracy despite its availability in long-term prediction. For this reason, it is the method that I will adopt in my study for electricity demand forecasting. The details of this approached is discussed in Section 4.2.

2.4 Research contribution

There has been a lot of research on planning and operating a specific type of energy storage system for electric grids, but a lack of research on the joint planning of energy storage and renewable energy sources. I focus on the latter, as I believe joint planning for energy storage along with renewable generation can potentially result in a more economical and efficient energy system. Moreover, the electricity consumption level and the availability of a renewable energy source (wind) vary in different geographical regions and at different times of the year. This makes the joint capacity optimisation for generation and storage very important when planning such power systems.

In this thesis, I consider the scenario of a stand-alone grid where there is high penetration of wind energy. I further assume that the grid has a fossil fuel generator (gas-fired plant), whose generation capacity is no more than the electricity consumption. The objective is to find the optimal values of the generator and storage capacities that minimise the associated total cost, subject to the constraint that the electricity demand is met at all times. The problem is studied by formulating it into a cost minimisation framework.

The main contributions of my study are as follows. In a first setting, I propose a basic model for an energy system that consists of a gas-fired plant and a small wind farm with a capacity for energy storage, where the demand and wind speeds are known and implement an optimisation procedure to find optimal values of the generator and storage capacities that minimise the associated total cost. In this way, the system as a whole becomes more efficient as the joint optimisation combines the benefits from each individual sub-system. The nature of my problem is constrained non-linear optimisation and it is solved in MATLAB.

I then gradually extend this deterministic model to study the stochastic modelling of the renewable energy source (wind) and electricity demand forecasting. Actual data is not always available or accessible. Therefore, it is necessary to apply models to forecast or simulate observations based on available historical data. Using synthetic data can also be useful to assess potential costs for a location where a wind farm is planned but does not exist yet. In this work, I use a MC model to capture the stochasticity of the wind speed, while for electricity demand, I use the mathematical model developed in (Filik et al., 2011) to construct a long-term forecasting model with hourly frequency. One of the main advantages of the selected method for electricity demand is that it is able to make short-, medium- and long-term hourly load forecasting within a single framework.

Furthermore, I include three model extensions to increase the flexibility of the developed model framework and improve the efficiency of the system. Firstly, a comparison of existing storage technologies further extends the model and assesses the cost-effectiveness of these different types of storage technologies. Secondly, I take into account the carbon emission modelling from the gas-fired plant by applying a carbon tax and a carbon emission constraint. Thirdly, I consider the possibility of connecting my system to the National Grid, which I import from, or export to, depending on whether my system has an energy shortage or surplus in meeting the demand.

My results could provide helpful insights in making decisions for planning a joint deployment of generation capacity and energy storage in future decentralised power grids with a high renewable penetration.

Chapter 3

Model framework

In this chapter, I first create a notation section, then describe the framework of the optimisation model in Section 3.2 and provide numerical examples to illustrate how it can be applied in practice in Section 3.3. Section 3.4 and 3.5 will contain the discussion of the relaxation of some of the constraints imposed.

3.1 Notation

This section contains all the variables and parameters used in Chapter 3. The three decision variables are differentiated with an overline, and those quantities that are a function of time are presented with a time index t. The time step in my framework is one hour, denoted by Δt.

t: Time index, $t = 1, 2, \ldots, n$.

Δt: Time step, $\Delta t = 1$

C_{inv}: Investment cost of the generator and storage

$C_{\mathrm{o/m}}$: O/M cost of the generator and storage

D_t: Electricity demand in time period t (MW)

f: Objective function for the optimisation problem

S_t: Energy in the storage minus the minimum level of energy in time period t (MWh)

S_t^+: Energy put into the storage in time period t (MWh)

S_t^-: Energy taken out of the storage in time period t (MWh)

$R_{\max}^+, R_{\max}^-$: Rated input and output power of the storage (MW)

$\overline{S_r}$: Energy capacity of the storage (MWh)

$S_{\min}, S_{\max}$: Lower and upper limit, respectively, of the energy capacity of the storage (MWh)

W_t: Electricity generation from wind in time period t (MWh)

$\overline{W_r}$: Maximum electricity generation from wind (MWh)

$W_{\min}, W_{\max}$: Lower and upper limit, respectively, of the electricity generation capacity from the wind turbine (MWh)

Y: Electricity generation from the gas-fired generator for each time period (MWh)

$\overline{Y_r}$: Maximum electricity generation of the gas-fired generator (MWh)

$Y_{\min}, Y_{\max}$: Lower and upper limit, respectively, of the generation capacity from the gas-fired generator (MWh)

P_g: Fuel price

Q: Fuel consumption

α: Conversion factor from megawatt hour to gigajoule

η: Fuel efficiency

δ: Rated energy/power ratio

ρ_R: Coefficient used to covert electricity to energy in the storage

ρ_E: Coefficient used to convert the energy in the storage to electricity

$\rho_R\rho_E$: Round-trip efficiency ($0<\rho_R\rho_E<1$)

3.2 Model

For simplicity, I consider only one type of fossil fuel generator - gas-fired plant, one type of renewable energy - wind, and one type of storage - battery energy storage. More specifically, I assume my system has one gas-fired plant and one small wind farm that consists of several wind turbines. The wind farm has one or more batteries attached. Figure 3.1 shows the modelled system. The objective is to find the optimal capacity of the gas-fired plant, the wind generator, and battery, such that electricity demand is met at every hour and the total of investment cost and operational/maintenance (O/M) cost is minimised. The basic model is deterministic, in other words, both wind speed and electricity demand within this setting would need to be known in advance. The stochasticity of wind speed and electricity demand forecasting will be addressed in the subsequent chapter.

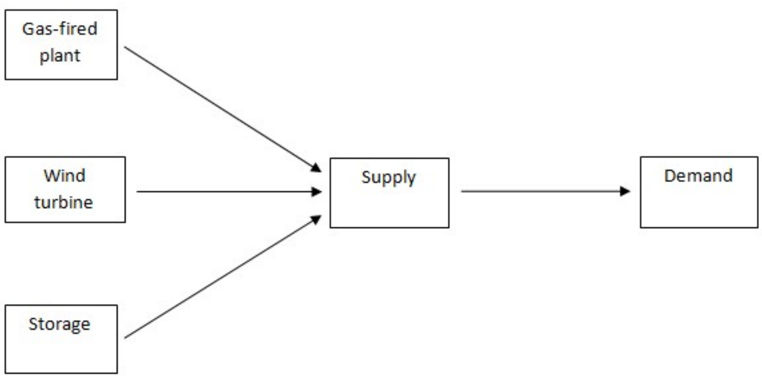

Figure 3.1: Diagram of the modelled system

Let me start with some technical assumptions for my basic model: (a) There is no import and export of energy. If the storage is full, then surplus energy will be discarded. (b) The gas-fired plant is not able to generate more than the electricity consumption and it has a constant energy output. (c) There is no ramping constraint for the gas-fired plant. (d) The charging and discharging process cannot take place simultaneously. Later, I will relax assumption (a) to increase the flexibility of the model in Chapter 5. The relaxation of the constraint on the gas-fired plant generation in assumption (b) is discussed in Section 3.4.

A. Generator model

The generators are classified into gas-fired generators and renewable generators. For the gas-fired generator, I assume it delivers a constant output (i.e. 20% of its full capacity), so that $Y_t = Y$. Its electricity output (Y) is non-negative and bounded above by the electricity consumption ($D_t \times \Delta t$), where Δt is the time step ($\Delta t = 1$ hour). Hence, for all $t > 0$,

$$0 \leq Y \leq D_t \times \Delta t.$$

The generation capacity of the gas-fired generator ($\overline{Y_r}$) is bounded by upper and lower limits,

$$Y_{\min} \leq \overline{Y_r} \leq Y_{\max}.$$

The cost of the gas-fired generator consists of the amortised investment cost , the O/M cost and the fuel cost. Fuel consumption (Q) is related to electricity output as follows:

$$Q = \frac{Y}{\alpha\eta}.$$

where η is the fuel efficiency parameter and α is the conversion factor, i.e., MWh (Megawatt hour) to GJ (Gigajoule).

The total cost of the gas-fired generator is,

$$C_{\text{gas}} = C_{\text{inv}}(\overline{Y_r}) + C_{\text{o/m}}(Y) + P_g \frac{Y}{\alpha\eta}.$$

In this thesis, I employ one type of renewable energy - wind. The power production from a wind turbine in a specific location depends on the cut-in, the rated and the cut-out wind speed. A turbine starts to generate power when the wind speed exceeds the cut-in speed. The power output gradually increases with the wind speed between the cut-in speed and rated wind speed, the power output remains constant at the rated power level until the cut-out speed, then the wind turbine stops for safety reasons (Nemes and Munteanu, 2011). Wind power is proportional to the cube of wind speed, wind energy production is also proportional to the cube of wind speed as energy (kWh) = power (kW) $\times$ time t ($t = 1$ hr).

Let W_t denote the electricity generation from the wind turbine during time period t, and $\overline{W_r}$ denote the maximum generation from the wind. The electricity production from a wind turbine is calculated using the following cubic function (Deshmukh and Deshmukh, 2008):

$$W_t = \begin{cases} 0, & \text{if } v_t < v_{in}, \\ \overline{W_r}\left(\frac{v_t - v_{in}}{v_r - v_{in}}\right)^3, & \text{if } v_{in} \leq v_t \leq v_r, \\ \overline{W_r}, & \text{if } v_r < v_t \leq v_{out}, \\ 0, & \text{if } v_t > v_{out}, \end{cases}$$

where

- v_t is the wind speed at time t;
- v_{in} is the cut-in wind speed [1];
- v_r is the rated wind speed [2];

[1] Wind turbine cannot rotate at very low wind speeds. As wind speed increases, the turbine begins to rotate and generate power. The cut-in speed is the minimum speed at which the turbine starts to rotate and generate power.

[2] As wind speed increases above the cut-in speed, the level of power generated increases very quickly. When the power output reaches to the maximum level, this speed is called the rated speed.

- $\overline{W_r}$ is the rated electricity production[3];
- v_{out} is the cut-out wind speed [4].

The generation capacity of a wind turbine is bounded by upper and lower limits,

$$W_{\min} \leq \overline{W_r} \leq W_{\max}.$$

The investment cost of a wind turbine depends on its generation capacity $\overline{W_r}$. The O/M cost is usually not evenly distributed over time as it tends to increase with time. Instead of letting it depend on how much electricity it produces, I express the O/M cost as a percentage of the investment cost. A wind turbine's O/M cost is usually a small percentage (i.e. 2%) of the investment cost in its early years. After six years, the O/M cost increases to approximately 5% of the total investment cost (Krohn et al., 2009; Liu et al., 2015). In this work, I am optimising over a one year period only, I assume that my turbines are at the early years of their lifetime. Therefore, the total cost of wind energy is

$$C_{\text{wind}} = (1+r) * C_{\text{inv}}(\overline{W_r}).$$

where the term $r * C_{\text{inv}}(\overline{W_r})$ denotes the O/M cost and r is the percentage of the investment cost (i.e. 2%).

B. Energy storage model

Suppose at time $t+1$, the electricity demand is D_{t+1}, the wind generation is W_{t+1} and the gas-fired plant generation is Y (it delivers a constant output so $Y_{t+1} = Y$). There are three scenarios to consider.

1. The total electricity generated exceeds the consumption, then the excess amount of electricity will be stored with a conversion factor ρ_R. The total amount of

[3]This is the maximum level of electricity that a turbine can generate. Even at higher speed, there is no further rise in the energy output.

[4]The cut-out speed is when the turbine is forced to shut down to avoid damage.

energy stored in the battery should not exceed its maximum energy capacity S_r. Once the storage is full, it stops charging and the surplus is discarded.

2. The total electricity generated is less than the consumption, but with the amount of energy available in the battery, it is enough to cover the shortage. The energy in the battery will be converted into electricity with a conversion factor ρ_E to fill the gap.

3. If the amount of electricity generated during the time interval $[t, t+1)$ plus the electricity that can be obtained in the battery is still not enough to satisfy the consumption, then the battery will be drained until it hits the minimum level of charge (i.e. 0 as S_t represents the amount of usable energy).

Motivated by the above, I propose the following form for the storage transition function ($\Delta t = 1$):

$$S_{t+1} = \begin{cases} \overline{S_r} & \text{, if } S_t + \rho_R(W_t + Y - D_t \times \Delta t) > \overline{S_r}, \\ S_t + \rho_R S^+_{t+1} & \text{, if } D_{t+1} \times \Delta t < W_{t+1} + Y, S_t + \rho_R S^+_{t+1} \leq \overline{S_r}, \\ S_t - \frac{1}{\rho_E} S^-_{t+1} & \text{, if } W_{t+1} + Y < D_{t+1} \times \Delta t < \rho_E S_t + W_{t+1} + Y, \\ 0 & \text{, otherwise .} \end{cases}$$

Once a charging/discharging process begins, I assume it takes place at a constant rate over that time period. For each time period t, the amount of power that flows into or out of the battery is limited by its rated power. In practice, the rated input power may be different from the output power; very often, people assume the equality of them (Sioshansi, 2011; Codemo et al., 2013; Garvey, 2015). For simplicity, I assume the equality of the two in the numerical examples. Let $R^+_{\max}$ denote the rated power, the maximum energy that can be put into the storage is $R^+_{\max} \times \Delta t$. Similarly, the maximum energy that can be taken out from the storage is $R^-_{\max} \times \Delta t$.

This leads to the following two limits,

$$S_t^+ \le R_{\max}^+ \times \Delta t,$$
$$S_t^- \le R_{\max}^- \times \Delta t.$$

The amount of energy can be charged (S_t^+) depends on how much energy has already been stored (S_{t-1}) and the energy capacity of the storage ($\overline{S_r}$), which cannot exceed its maximum allowed amount ($R_{\max}^+ \times \Delta t$). Similarly, the amount of energy discharged (S_t^-) cannot exceed its maximum allowance ($R_{\max}^- \times \Delta t$) and the amount of energy available in the storage (S_{t-1}). Hence, S_t^+ and S_t^- are defined as:

$$S_t^+ = \max(\min(W_t + Y - D_t \times \Delta t, (\overline{S_r} - S_{t-1})/\rho_R, R_{\max}^+ \times \Delta t), 0),$$
$$S_t^- = \max(\min(D_t \times \Delta t - W_t - Y, \rho_E S_{t-1}, R_{\max}^- \times \Delta t), 0)$$

In this thesis, I let δ denote the ratio between the rated energy and the rated power (i.e. duration). Batteries and flywheels are able to get fully charged within one hour ($\delta = 1$). For compressed air energy storage and pumped electric storage, they might take ten hours to be charged ($\delta = 10$). Therefore, $R_{\max}^+ = \overline{S_r}/\delta$ and $R_{\max}^- = \overline{S_r}/\delta$. With a storage capacity of 10 MWh, batteries and flywheels can take in 10 MWh of energy in one hour, but only 1 MWh can be absorbed for compressed air energy storage and pumped electric storage. I assume the equality of the rated input and output power, if one has appropriate information about them, the charging/discharging capacity can be modified easily.

As before, upper and lower limits are imposed on the energy capacity for the battery ($\overline{S_r}$),

$$S_{\min} \le \overline{S_r} \le S_{\max}.$$

The cost of the battery storage including the investment cost and the O/M cost is:

$$C_{\text{battery}} = C_{\text{inv}}(\overline{S_r}) + C_{\text{o/m}}(S_t^+, S_t^-).$$

In this equation, the O/M cost depends on the amount of energy charged or discharged.

C. Demand constraint

The total generation from the gas-fired plant, the wind turbine and the amount of energy in the storage should be equal to or greater than the total electricity consumption. I write the demand constraint as follows:

$$D_t \times \Delta t \leq Y + W_t + S_t^-, \forall t$$

where the electricity demand D_t is measured in MW and electricity generation is measured in MWh, so I need to multiply the demand D_t by Δt ($\Delta t = 1$).

D. Optimisation problem

The objective is to find the optimal generation capacities of the gas-fired plant and the wind turbine, and the optimal size of the battery, such that the total of the investment cost and the O/M cost over a period of n time periods is minimised, where $n = 8736$ hours (i.e. 364 days), subject to the constraint that electricity demand is met in every hour. The year 2012 has 366 days in total, the reason for 364 days is because the demand used for the prediction of 2012 is year 2011, which has 365 days only, and there is a missing wind speed value for the 2012 dataset so this particular day is excluded. Therefore, a total of 2 days fewer leads to 364 days instead of 366 days.

The variables to be optimised are the capacities of the gas-fired plant, the wind turbine and the battery storage, denoted by $(\overline{Y_r}, \overline{W_r}, \overline{S_r})$. Let me define the objective function to be:

$$\begin{aligned} f\left(\overline{Y_r}, \overline{W_r}, \overline{S_r}\right) := &\sum_{t=1}^{n} \left(C_{\text{o/m}}(Y) + P_g \frac{Y}{\alpha\eta} + C_{\text{o/m}}(S_t^+, S_t^-)\right) \\ &+ C_{\text{inv}}(\overline{Y_r}) + (1+r)C_{\text{inv}}(\overline{W_r}) + C_{\text{inv}}(\overline{S_r}), \end{aligned} \tag{3.1}$$

where $r * C_{\text{inv}}(\overline{W_r})$ denotes the O/M cost for the wind turbine.

The optimisation problem is

$$\min f\left(\overline{Y_r}, \overline{W_r}, \overline{S_r}\right),$$

subject to

$$Y_{\min} \leq \overline{Y_r} \leq Y_{\max}, W_{\min} \leq \overline{W_r} \leq W_{\max}, S_{\min} \leq \overline{S_r} \leq S_{\max}, \tag{3.2}$$

$$0 \leq Y \leq D_t \times \Delta t, D_t \times \Delta t \leq Y + W_t + S_t^-, t = 1, \ldots, n. \tag{3.3}$$

Here in (3.2) I assume box constraints on the capacities, and the constraints in (3.3) impose respectively that the gas-fired plant cannot generate more than the electricity consumption, and that the consumption must be covered at all times by a combination of the electricity generation from the gas-fired plant, the wind turbine and the amount of energy in the battery storage. Note that, the demand D_t is measured in MW and electricity generation is measured in MWh, so the constraints in (3.3) make sense when I multiply the demand D_t by Δt ($\Delta t = 1$).

3.3 Numerical example

Using real data, I provide numerical examples to show how the proposed framework can be applied in practice to make decisions on joint renewable generation and energy storage planning. The numerical examples are also contained in Appendix B.1.

The geographic location has a significant impact on the availability of wind energy. I choose two sites, one in Aberdeenshire city (I refer as Aberdeen throughout the thesis) and one in Rugby, with completely different wind characteristics, in order to make a comparison between the optimal solutions obtained. Aberdeen is situated in Scotland and it is windy all year round, whereas, Rugby is located in the West Midlands with weaker wind conditions. The two chosen sites for my numerical examples are two extreme cases. Given a similar level of demand, if the same types of turbines are installed regardless the wind condition, then the optimal solutions would differ significantly. For a site with wind conditions in between, one would expect the optimal solution to lie in-between the solutions from these two extreme cases. However, if different classes of turbines are installed based on the wind condition of each site, then one would expect the difference between the optimal solutions to be smaller. Based on the wind speed of the site, wind turbines are selected to maximise the energy production.

3.3.1 Data collection

1. Hourly wind speed data
2. Hourly electricity demand data
3. Investment and O/M cost of the wind turbine, the battery storage and the gas-fired plant
4. Fuel price, fuel efficiency and conversion factor for the gas-fired plant
5. Efficiency and duration of the battery storage
6. Economic lifetime of the wind turbine, the battery storage and the gas-fired plant

All data used for the numerical examples are over a one year period, i.e., 1 January to 31 December 2012. The half-hourly electricity demand data at a national level are obtained from the National Grid website under data explorer - historical demand data, and the annual sub-national electricity consumption data are obtained from the Department of Energy and Climate Change website (named sub-national electricity consumption statistics). I first take a sum of the half-hourly demand to obtain hourly demand then scale down by a factor to represent the hourly demand at a regional level (Aberdeen and Rugby). Hourly wind speed data from the Met office are downloaded from the British Atmospheric Data Centre. Wind speed was originally measured in knots then converted into m/s.

Due to the limited size of my system, it is only able to supply a proportion of the total electricity consumption. The proportion is chosen based on the examination of the electricity generation from the wind, the gas-fired plant and the storage. The chosen proportion of the consumption must be appropriate, otherwise, the algorithm does not output a solution. I consider scaling the consumption to be in the range of 2.4 - 4.5 MWh.

According to the International Electrotechnical Commission (IEC) standard, there are 4 classes of wind turbines. Wind class I turbines are designed for sites with average wind speeds over 8.5 m/s; wind class II turbines are for sites with wind speeds up to 8.5 m/s on average; wind class III turbines are designed for sites with average wind speeds up to 7.5 m/s and wind class IV turbines apply to sites with average wind speed below 6 m/s. Based on the examination of the obtained wind speed data for the year 2012, wind class I turbines should be used in Aberdeen and wind class IV turbines should be used in Rugby. Referring to the book written by Freris and Infield (2008) - Renewable Energy in Power Systems, the average capacity factor of onshore wind turbines is around 30% in the UK. Therefore, I make the assumption that the wind turbines used for both locations have similar capacity factors.

Wind turbine class	I	IV
Cut-in wind speed (m/s)	3	2.5
Cut-out wind speed (m/s)	25	16
Rated wind speed (m/s)	13.5	6.8

Table 3.1: The cut-in, cut-out and rated speed for class I & IV turbine

Information on gas-fired plants are found in (Brinckerhoff, 2004; IEA, 2010a,b; West, 2011; Kelp et al., 2014). For a typical open cycle gas turbine, its hourly generation is 10 - 300 MWh when running at full capacity. In the numerical examples, I assume that the gas-fired plant generates 20% of its full capacity in each period. So, a gas-fired plant with a capacity of $\overline{Y_r}$ will generate $0.2\overline{Y_r}$ MWh of electricity for every hour. The investment cost of a gas-fired plant is £0.7 million per MWh. A gas-fired plant has an economic life of 25 years. The O/M cost is £3.4 per MWh on average. The quarterly price of natural gas for the year 2012 is obtained from the statistical data set named average prices of fuels purchased by the major UK power producers (QEP 3.2.1) from the Department of Energy and Climate Change website. The fuel efficiency is $\eta = 0.6$, and the conversion factor from MWh to GJ is $\alpha = 0.278$.

For wind turbines, all the relevant information is obtained from (Brinckerhoff, 2004; McGowin, 2007; Krohn et al., 2009; IEA, 2010b; West, 2011; Yang and Nehorai, 2014; Kelp et al., 2014). The investment cost of a wind turbine is about £1.6 million per MWh and a typical turbine lifetime is 20 years. As the O/M cost for a wind turbine has a strong time component even if it has not operated at all, assuming the turbine is in the early years of its lifetime, I set the O/M costs to be 2% of the total investment cost. I am optimising over a one year time period only.

Characteristics and cost information on advanced lead-acid batteries are taken from (Gönen, 2011; Komor and Glassmire, 2012; Battke et al., 2013; Carnegie et al., 2013; Akhil et al., 2013; Luo et al., 2015). The typical cycle life for lead-acid batteries is 500 - 1000 cycles and the cycle life of advanced lead acid batteries can be up to 2000 cycles. The designed calendar lifetime is 10 - 15 years on average, but it strongly depends on how often it completes a cycle. If a battery only completes 40 full cycles per year, and its cycle life is 600 cycles, then it is expected to last 15 years. In my system, the time it takes to complete a cycle is fairly long and the depth of discharge is relatively low, so I expect the battery storage to have a lifetime of 15 years. The investment cost is £2.2 million per MWh and the O/M cost is around £3.6 per MWh. Conversion factors ρ_E and ρ_R are set as 0.95, which makes a round-trip efficiency of $\rho_E \rho_R = 0.9$. The starting storage level $R_1 = 0$. The time duration for advanced lead-acid batteries is 1 hour (i.e. $\delta = 1$). Although people often make the assumption that an energy storage has the same charging and discharging capacity, it is practical to have different power ratings for power input and output (Sioshansi, 2011; Codemo et al., 2013; Garvey, 2015). For simplicity, I assume equality of the two in this thesis. However, if necessary, this can be easily modified when one has such information on the different rated input and output power.

I use linear cost functions in the numerical examples and assume that the investment cost can be spread over the lifetime of a plant. Of course, if one has more detailed cost information, then it will help to work out more appropriate forms for the cost functions, and this can be easily modified in the model. The investment and O/M cost are given in Table 3.2. The O/M cost for a gas-fired plant does not include the fuel cost (it is calculated separately) and the O/M cost for a wind turbine is set to be 2% of its investment cost.

Type	Investment cost (M£/MWh)	Life span (years)	O/M cost (£/MWh)
Gas-fired plant	0.7	25	3.4
Wind turbine	1.6	20	2% of its investment cost
Battery	2.3	15	3.6

Table 3.2: Information on generators and energy storage

I consider the following linear cost functions,

$$C_{\text{inv}}(\overline{Y_r}) = a_1 \overline{Y_r}, \quad C_{\text{inv}}(\overline{W_r}) = a_2 \overline{W_r}, \quad C_{\text{inv}}(\overline{S_r}) = a_3 \overline{S_r},$$
$$C_{\text{o/m}}(Y) = b_1 Y, \quad C_{\text{o/m}}(S_t^+, S_t^-) = b_2 S_t^+ \text{ or } b_2 S_t^-.$$

where a_1, a_2, a_3, b_1 and b_2 are the corresponding cost coefficients shown in Table 3.2. That is $a_1 = 28000, a_2 = 80000, a_3 = 153333, b_1 = 3.4, b_2 = 3.6$.

The charging and discharging process do not take place at the same time, so $C_{\text{o/m}}(S_t^+, S_t^-)$ is either $b_2 S_t^+$ or $b_2 S_t^-$ depending on whether it is charging or discharging for that time period. I set $[Y_{\min}, Y_{\max}] = [10, 12]$, $[W_{\min}, W_{\max}] = [10, 100]$ and $[S_{\min}, S_{\max}] = [30, 100]$.

3.3.2 Results and discussion

The optimisation problem, as a constrained non-linear minimisation of multiple variables, is solved in MATLAB with a gradient-based method for finding local minima with iterations. It starts with an initial guess, iterates according to a given update scheme and then finishes upon a stopping criterion is met. A local optimum is determined if the first order necessary and second order sufficient conditions are satisfied. Different initial starting points are used to look for a global optimum.

Let $(\overline{Y_r}^*, \overline{W_r}^*, \overline{S_r}^*)$ denote the vector of optimal solutions, measured in MWh.

Type	Aberdeen	Rugby
Gas-fired plant $\overline{Y_r}^*$	12	12
Wind turbine $\overline{W_r}^*$	10.04	18.60
Battery $\overline{S_r}^*$	30	30.03

Table 3.3: Optimal capacities for energy storage and generators

With the above choice of parameter values, I implement the optimisation problem with different initial starting points, which returns an optimal capacity $(\overline{Y_r}^*, \overline{W_r}^*, \overline{S_r}^*) = (12, 10.04, 30)$ with a total cost of £6.57 million for Aberdeen for the year 2012. The optimal capacity is $(\overline{Y_r}^*, \overline{W_r}^*, \overline{S_r}^*) = (12, 18.60, 30.03)$ and the total cost over the year 2012 is £7.28 million for Rugby.

It is noted that the optimal capacities for the gas generator and storage are similar for the two locations. The wind turbine capacity is different. In this system, the gas-fired plant is the only reliable supply in the system and it is cheaper. With the objective being minimising cost, the gas-fired plant will be selected to generate its maximum allowed amount. Aberdeen has a higher mean wind speed and it is more volatile. The highest wind speed is 35 m/s in Aberdeen, while it is just over 11 m/s in Rugby (see Figure 3.2). With such a big difference in the wind speed, different classes of turbines should be installed for each site to maximise the wind energy production. Class I turbines are used for Aberdeen and class IV turbines are used for Rugby. Moreover, the average hourly electricity consumption over the year may roughly be the same, but the pattern may differ. It is expected to see a difference in the wind turbine capacity. The size of the storage is similar for both locations.

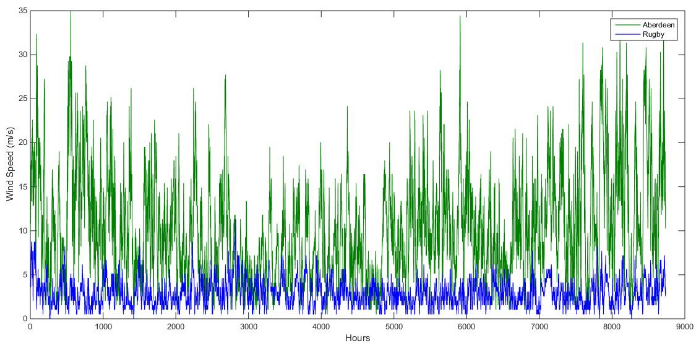

Figure 3.2: Wind speed of Aberdeen against wind speed of Rugby of year 2012

To seek for a global optimum, I optimise using different initial starting points during implementation, and I have chosen the solution that yields the lowest cost. Therefore, I believe this local minimum is a global minimum. The objective functional in my problem is a monotonically increasing function, so the solution should be around the lower bound. I present the surface plot with the variable $\overline{Y_r}$ being fixed in Figure 3.3 using the Aberdeen example. Looking at the plot, it is obvious that the the minimum is achieved at the lower corner, the obtained solution should be a global minimum.

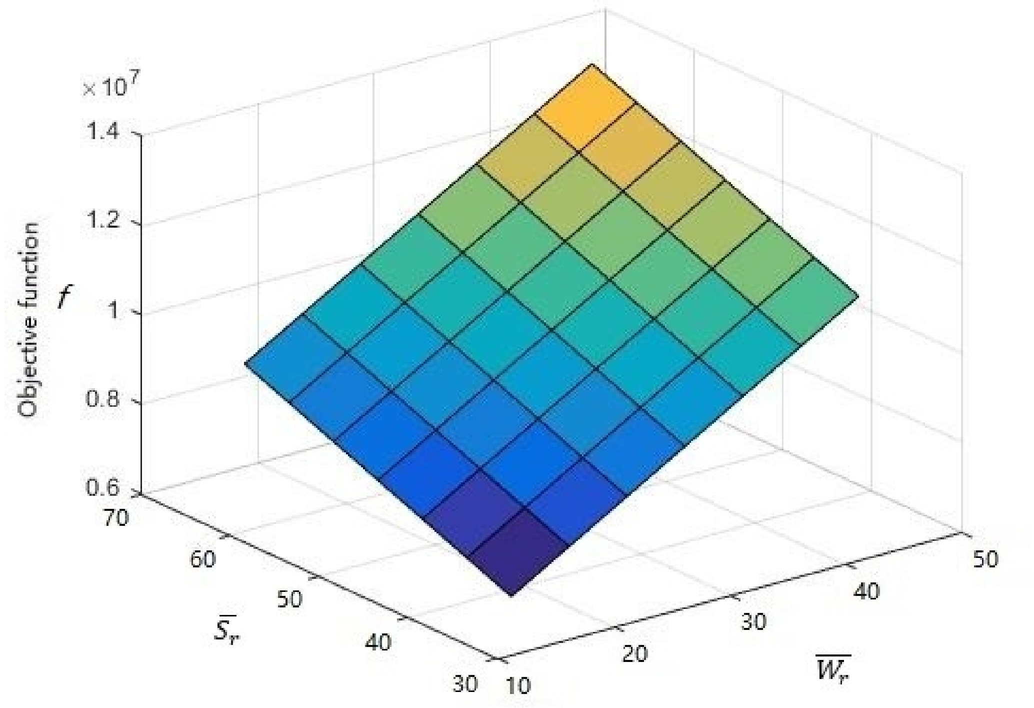

Figure 3.3: Surface plot of the objective function

3.4 Relaxation of the constraint on the gas-fired plant generation

The original problem contains 3 variables and is optimised over 8736 time periods; this is too complex to compute by hand. I have shown the effect by running some numerical simulations. The constraint on the gas-fired plant generation is $Y \leq D_t \times \Delta t$, this is equivalent to $\overline{Y_r} \leq 12$ as $Y = 0.2\overline{Y_r}$ and $\max(Y) = \min(D_t) = 2.4$. By relaxation, I meant that it could have $\overline{Y_r} > 12$.

The objective functional is monotonically increasing, thus, the solution is very likely to be close to the lower boundary. Before relaxation, the lower and upper bounds are [10, 10, 30] and [12, 100, 100] for Aberdeen, the optimal solution is (12, 10.04, 30) with a cost of £6.57 million. Now, it can have $\overline{Y_r} > 12$, using the same lower bound and increase the upper bound of the gas-fired plant capacity to 100, I run the model, which generates an optimal solution of (12.0086, 10, 30) with a cost of 6.57 million. The gas-fired plant capacity increased by 0.0086 while the wind turbine capacity reduced to its lower bound. As the change in the solution is minimal, I further reduce the lower bound of the capacity for the wind turbine and the storage, setting the lower bound to be [10, 1, 1] and the upper bound to be [100, 100, 100], I re-run the model, it generates an optimal solution of (19, 1, 1) with a cost of 2.06 million.

The relaxation of the constraint on the gas-fired plant generation allows the gas-fired plant to generate as much as it likes. To meet the same level of electricity demand with a greater supply from the gas-fired plant, it does not rely on the wind and the storage as much as before. By allowing a greater supply from the gas-fired plant, it will be cost-saving due to cheaper investment cost. In this thesis, I study an independent grid where the penetration of wind energy is high. Therefore, I would like the majority of supply to come from the wind rather than the gas-fired plant. For this purpose, I impose a constraint to limit the gas-fired plant generation. To meet the aggressive carbon emission target, the government encourages the use of renewable energy. Meanwhile, the cost of wind farms continues to fall. By 2025, it is projected that the total installed costs of onshore wind farms would decline by 12% (Amin, 2016). By then, wind energy would be more competitive to fossil fuel plants.

3.5 Loss of load

The loss of load has not been considered as I imposed the constraint that electricity demand must be met at every one hour. In this section, I study the loss of load by introducing a penalty function. The goal of a penalty function is to add a term that prescribes a high cost for constraint violation and it works in the following way. Suppose there is a free highway that monitors when drivers enter and exit the road. The rules are: you can drive as fast as you like; when you drive under 80 mph then it is free; for every mph you drive over 80, it costs £5000. In this particular example, there is no constraint as people can drive as fast as they like. But the effect of these rules would still limit driving speed to 80 mph.

For simplicity, I study the problem by looking at a single time period. Considering a time period with an electricity consumption of 2.4 MWh and a wind speed of 16 m/s. With the constraint $2.4 \leq W + Y + S^{-}$, I need a penalty that is zero if $2.4 - W - Y - S^{-} \leq 0$, or positive if $2.4 - W - Y - S^{-} > 0$. Therefore, for violating the constraint, I add the penalty function $\kappa \cdot \max\left(0, (2.4 - W - Y - S^{-})\right)$. So the objective function becomes $\tilde{f} = f + \kappa \cdot \max\left(0, (2.4 - W - Y - S^{-})\right)$. The idea is that for a small value of κ, it may choose not to satisfy the constraint, but with a large value of κ, it will try to satisfy the constraint. Set $[Y_{\text{min}}, Y_{\text{max}}] = [1, 12]$, $[S_{\text{min}}, S_{\text{max}}] = [1, 10]$ and $[R_{\text{min}}, R_{\text{max}}] = [1, 10]$. I run the model with $\kappa = 1$, and obtained an optimal solution of $(\overline{Y_r}^*, \overline{W_r}^*, \overline{S_r}^*) = (1, 1, 1)$ and the total cost is £39.04. It is obvious that the constraint is not satisfied. I keep increasing the penalty value until $\kappa = 10$, the optimal solution becomes $(\overline{Y_r}^*, \overline{W_r}^*, \overline{S_r}^*) = (1.0022, 2.1996, 1.0011)$ and the total cost is now £49.04. The constraint is now satisfied due to a higher penalty.

When there are more than one time period, the penalty function is based on the number of constraints violated. The objective function would be $\tilde{f} = f + \sum_{t=1}^{n} \kappa \cdot \max\left(0, (D_t \times \Delta t - W_t - Y - S_t^{-})\right)$. I now run a simulation for a total of five time periods.

Consumption (MWh)	2.4	2.7	2.6	2.7	2.5
Wind speed (m/s)	16	17	24	20	19

I start with $\kappa = 1$ and increase the value by 1 for each simulation. Let μ

denote the no. of constraints violated. The results are presented as follows:

κ	$(\overline{Y_r}^*, \overline{W_r}^*, \overline{S_r}^*)$	$\tilde{f}$	μ
1	(1, 1, 1)	£196.11	5
10	(1.0004, 2.1999, 1.0002)	£254.05	4
12	(1, 2.3, 1)	£255.66	3
17	(1, 2.4, 1)	£258.07	2
20	(1, 2.4526, 1)	£258.61	1
51	(1, 2.5, 1)	£260.04	0

With a small κ, for example, $\kappa = 1$, the constraint is not satisfied for all five time periods. As κ increases gradually, the no. of constraint violations decreases. When κ increases to 51, the constraint is now satisfied for all five time periods.

In conclusion, for an optimisation problem with n time periods (n is large), the value of κ would need to be very high for the constraint to be satisfied for most (if not all) of the time periods. A time period with a shortage would cause a blackout and the shortage would need to be bought from elsewhere. In Chapter 5, I will look at the possibility of connecting the network with the National Grid, where I can import from, or export to, depending on whether I have a surplus or shortage.

Chapter 4

Stochastic modelling of wind speed and electricity demand forecasting

In this chapter, I study the stochasticity of wind speed and electricity demand forecasting. Wind speed is captured using a MC model, and electricity demand is predicted using the method proposed by Filik et al. (2011), which attempts to make long-term forecasting with an hourly accuracy.

4.1 Wind speed

It is well known that wind is a highly variable resource and its variability can be observed both spatially and temporally. The analysis is based on 1-year observed hourly wind speed time series of the year 2012 for Aberdeen and Rugby.

Even though the Weibull distribution is commonly used for the frequency analysis of wind speed data, it does not take into account of the dependency over time in the wind speed time series. In my system, the use of energy storage makes it necessary to capture the time correlation of the wind speeds, since the storage is intimately linked to the generation from wind. MC models have been frequently used for the generation of synthetic wind speed series (Carpinone et al., 2010), and it is particularly suited for modelling systems where the current state of a sequence is highly correlated to the state immediately preceding it (McLoughlin et al., 2010). MC modelling is based on the construction of a transitional probability matrix where the transition from one discrete to another discrete state is represented in terms of its probability.

A MC can be described as a stochastic process which a state changes between discrete time steps, and it can be parameterised by estimating the transition probabilities between the states in the observed systems (Balzter, 2000). More precisely, a MC is a sequence of random variables $\{X_t\}_{t\geq 0}$ such that, for any $t = 0, 1, 2, \ldots$, the random variable $\{X_t\}$ can take values from a discrete set of states $\{i_0 \ldots, i_N\}$ and satisfies the following Markov property for conditional probabilities:

$$
\begin{aligned}
&P(X_{t+1} = i_{t+1} \mid X_t = i_t, X_{t-1} = i_{t-1,\ldots,} X_0 = i_0) \\
&= P(X_{t+1} = i_{t+1} \mid X_t = i_t).
\end{aligned}
$$

In other words, the probability of the state at time $t+1$ depends only on the state at time t. Denote $P(X_{t+1} = j \mid X_t = i) = p_{ij}$ for all states i and j. The transition probability matrix (TPM) can be constructed by using the following formula (Karatepe and Corscadden, 2013),

$$p_{ij} = \frac{n_{ij}}{\sum_{j=1}^{s} n_{ij}}.$$

where n_{ij} represents the number of transitions from state i to state j during one period. The TPM for a first order MC with s states can be written as

$$
P = \begin{bmatrix}
p_{1,1} & p_{1,2} & \cdots & p_{1,s} \\
p_{2,1} & p_{2,2} & \cdots & p_{2,s} \\
\vdots & \vdots & \ddots & \vdots \\
p_{s,1} & p_{s,2} & \cdots & p_{s,s}
\end{bmatrix}
$$

According to the definition, $p_{ij} \geq 0$, and $\sum_{j=1}^{s} p_{ij} = 1$ for all $i, j = 1, \ldots, s$.

The cumulative probability matrix (CPM) $C = [C_{ik}]$ can be constructed according to (Karatepe and Corscadden, 2013) as:

$$C_{ik} = \sum_{j=1}^{k} p_{ij}.$$

Now I provide an example of calculating the transition matrix and cumulative matrix for a first order MC model. Assume there is a MC of 10 wind speed values and it is divided into 2 states (state 1 contains wind speed no greater than 5 m/s and state 2 contains wind speed no greater than 10 m/s), then I should obtain a 2×2 transition matrix.

Wind speed (m/s)	3	2	2	6	7	9	7	8	1	4
State	1	1	1	2	2	2	2	2	1	1

The number of transitions from state 1 and remain in state 1 is 3 and the number of transitions from state 1 to state 2 is 1. Therefore,

$$p_{11} = \frac{3}{4} = 0.75, \quad p_{12} = \frac{1}{4} = 0.25.$$

Similarly, I count that the number of transitions from state 2 to state 1 is 1 and the number of transitions from state 2 and remain in state 2 is 4. This gives

$$p_{21} = \frac{1}{5} = 0.2, \quad p_{22} = \frac{4}{5} = 0.8.$$

The transition matrix is

$$P = \begin{bmatrix} 0.75 & 0.25 \\ 0.2 & 0.8 \end{bmatrix}$$

By adding the transition probabilities in each row, I have

$$c_{11} = 0.75, \quad c_{12} = 0.75 + 0.25 = 1,$$
$$c_{21} = 0.2, \quad c_{22} = 0.2 + 0.8 = 1.$$

Hence, the cumulative matrix is

$$C = \begin{bmatrix} 0.75 & 1 \\ 0.2 & 1 \end{bmatrix}$$

A second order MC is defined similarly, i.e., it requires

$$\begin{aligned} &P(X_{t+1} = i_{t+1} \mid X_t = i_t, X_{t-1} = i_{t-1,\ldots,}X_0 = i_0) \\ &\quad = P(X_{t+1} = i_{t+1} \mid X_t = i_t, X_{t-1} = i_{t-1}). \end{aligned}$$

Denoting $P(X_{t+1} = k \mid X_t = j, X_{t-1} = i) = p_{ijk}$ for all states i, j and k, I can write a second order TPM as

$$\tilde{P} = \begin{bmatrix} p_{1,1,1} & p_{1,1,2} & \cdots & p_{1,1,s} \\ p_{1,2,1} & p_{1,2,2} & \cdots & p_{1,2,s} \\ \vdots & \vdots & \ddots & \vdots \\ p_{1,s,1} & p_{1,s,2} & \cdots & p_{1,s,s} \\ p_{2,1,1} & p_{2,1,2} & \cdots & p_{2,1,s} \\ \vdots & \vdots & \ddots & \vdots \\ p_{s,s,1} & p_{s,s,2} & \cdots & p_{s,s,s} \end{bmatrix}$$

Similarly, then the CPM $C = [C_{ijk}]$ can be calculated according to (Karatepe and Corscadden, 2013) as follows:

$$C_{ijk} = \sum_{l=1}^{k} p_{ijl}.$$

Here, I give an example of calculating the TPM and CPM for using the same data points from previous example. For a second order Markov chain model, the transition matrix should be 4×2.

$$p_{111} = \frac{1}{2} = 0.5, \quad p_{112} = \frac{1}{2} = 0.5, \quad p_{121} = \frac{0}{1} = 0, \quad p_{122} = \frac{1}{1} = 1,$$
$$p_{211} = \frac{1}{1} = 1, \quad p_{212} = \frac{0}{1} = 0, \quad p_{221} = \frac{1}{4} = 0.25, \quad p_{222} = \frac{3}{4} = 0.75.$$

The transition matrix is

$$\tilde{P} = \begin{bmatrix} 0.5 & 0.5 \\ 0 & 1 \\ 1 & 0 \\ 0.25 & 0.75 \end{bmatrix}$$

For the cumulative probabilities, I have

$$\begin{aligned} c_{111} = 0.5, \quad c_{112} = 0.5 + 0.5 = 1, \\ c_{121} = 0, \quad c_{122} = 0 + 1 = 1, \\ c_{211} = 1, \quad c_{212} = 1 + 0 = 1, \\ c_{221} = 0.25, \quad c_{222} = 0.25 + 0.75 = 1. \end{aligned}$$

Hence, the corresponding cumulative matrix is

$$\tilde{C} = \begin{bmatrix} 0.5 & 1 \\ 0 & 1 \\ 1 & 1 \\ 0.25 & 1 \end{bmatrix}$$

The MC simulation procedure for synthetic generation of wind speed times-series is accomplished using the following steps (Karatepe and Corscadden, 2013):

1. Define the states of the MC and construct the TPM from the available data.

2. Construct the CPM.

3. Generate uniformly distributed random numbers between 0 and 1.

4. Select an initial state i.

5. Compare the value of the generated random number with the elements in ith row of the CPM. The next state is determined as follows: the value of the random number is greater than the cumulative probability from previous

state but no more than that of the following state, say j. Then the next random number needs to be compared with the elements from the jth row of the CPM.

6. A transition from state i to state j in the CPM can be converted into wind speed by $Z = Z_{j-1} + V_i(Z_j - Z_{j-1})$, where Z_{j-1} and Z_j are the lower and upper boundaries of the state, V_i is the random number.

For validation of the model, I use a combination of visual evaluation and comparison of statistical properties, which consist of descriptive statistics, Weibull distribution parameters, the probability distribution and the autocorrelation functions (ACF) of the wind speed time series. The ACF refers to the correlation of a time series with its own past and future values, and it is used to determine the persistence structure in the wind speed time series. The autocorrelation at lag time lag k can be determined using the following equation (Shamshad et al., 2005):

$$L_k = \frac{\frac{1}{N-k}\sum_{i=1}^{N-k}(v_i - \bar{v})(v_{i-k} - \bar{v})}{\frac{1}{N}\sum_{i=1}^{N}(v_i - \bar{v})^2},$$

where $\bar{v}$ is the mean of wind speed time series ($v_i, i = 1, 2, \ldots, N$).

Wind speed and wind power time series often contain the value of zero wind speed or power; when there is the existence of zero, I present the simulation error as the difference between the actual and the simulated values (say d), otherwise, I calculate the signed relative error which is the difference d divided by the actual value.

4.1.1 Wind speed of Aberdeen

Hourly wind speed data of the year 2012 for Aberdeen is shown in Figure 4.1. The highest wind speed is 34.95 m/s and the lowest speed is 0.514 m/s.

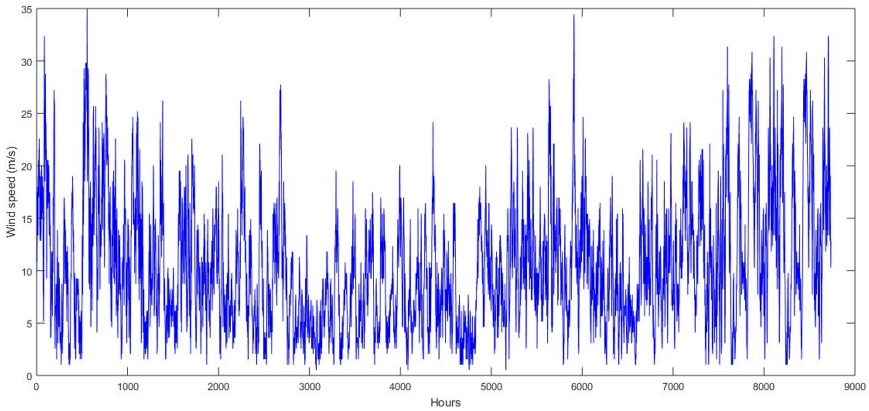

Figure 4.1: Hourly wind speed of year 2012

To apply a MC model, I first divide the wind speed into states (see Table 4.1). There are several ways to determine the number of wind speed states. In the study carried by Sahin and Sen (2001), states are defined based on the standard deviation of the dataset, so each state is taken as wide as one standard deviation of the observed hourly wind speed time series. In another study, Dukes and Palutikof (1995) use a fixed width for the states, which equals to 2 m/s. In my study, I divide the wind speed data set into 35 states for Aberdeen. Based on the visual examination of the histogram of the wind speed, I assign an upper and lower limit which has a difference of 1 m/s for the wind speed states. These upper and lower limits are highly subjective values (Aksoy et al., 2004).

Wind speed (m/s)	State	Wind speed (m/s)	State	Wind speed (m/s)	State
$0.5 \leq v \leq 1.5$	1	$12.5 < v \leq 13.5$	13	$24.5 < v \leq 25.5$	25
$1.5 < v \leq 2.5$	2	$13.5 < v \leq 14.5$	14	$25.5 < v \leq 26.5$	26
$2.5 < v \leq 3.5$	3	$14.5 < v \leq 15.5$	15	$26.5 < v \leq 27.5$	27
$3.5 < v \leq 4.5$	4	$15.5 < v \leq 16.5$	16	$27.5 < v \leq 28.5$	28
$4.5 < v \leq 5.5$	5	$16.5 < v \leq 17.5$	17	$28.5 < v \leq 29.5$	29
$5.5 < v \leq 6.5$	6	$17.5 < v \leq 18.5$	18	$29.5 < v \leq 30.5$	30
$6.5 < v \leq 7.5$	7	$18.5 < v \leq 19.5$	19	$30.5 < v \leq 31.5$	31
$7.5 < v \leq 8.5$	8	$19.5 < v \leq 20.5$	20	$31.5 < v \leq 32.5$	32
$8.5 < v \leq 9.5$	9	$20.5 < v \leq 21.5$	21	$32.5 < v \leq 33.5$	33
$9.5 < v \leq 10.5$	10	$21.5 < v \leq 22.5$	22	$33.5 < v \leq 34.5$	34
$10.5 < v \leq 11.5$	11	$22.5 < v \leq 23.5$	23	$34.5 < v \leq 35.5$	35
$11.5 < v \leq 12.5$	12	$23.5 < v \leq 24.5$	24		

Table 4.1: No. of states of the wind speed data

Due to the large size of the first order TPM (35×35) and the even larger size of the second order TPM (1225×35), these are not shown in the thesis. The first order TPM reveals that the highest probability occurs on the diagonal. In other words, if the current wind speed is known, the next wind speed is most likely to be in the same state as the current wind speed. Furthermore, the probability mass is concentrated around the diagonal and this implies that the probability of a transition between far states is infrequent. By examining the second order TPM, it shows that the highest probability appears on the diagonal, in other words, if the current and proceeding wind speeds are known then it is highly likely the next wind speed will lie in the same category.

Descriptive statistics and the Weibull distribution parameters of observed and simulated wind speeds are presented in Table 4.2. It is clear that first and second order MC models are sufficient to preserve most of the parameter values. As expected, a second order MC model shows improvement in the performance. Figure 4.2 and 4.4 represent the actual and synthetic data of the first order and second order MC model, respectively. Very often, wind speed time series contain the value of zero wind speed; I present the simulation error as the difference between the actual and the simulated values (Figures 4.3, 4.5). The values of the error terms seem to be smaller for the second order MC model. Thus, a second order MC model produces more satisfying results.

The probability distribution and the ACF plot of the observed and synthetic wind speed data are shown in Figure 4.6 and 4.7. For the probability distribution, the visual examination of the bars in this figure reveals that the probability at different wind speed time series almost has the same values. The probability distribution of both observed and synthetic data are characterised by the Weibull distribution. However, the curve of the synthetic data by a second order MC is closer to the curve of the actual data. From the ACF plot, it is clear that the data is not random as the autocorrelations are non-zero. This suggests that the MC models do produce some dependence structure of the time series. The general behaviour of the autocorrelation functions is similar for the two MC models at the start. As the order of time lag increases, a second order MC model performs slightly better than a first order MC model.

	Actual	1st order MC	2nd order MC
Mean	10.41	10.33	10.46
Std deviation	6.01	6.31	5.88
Variance	36.16	39.81	34.57
Median	9.25	8.99	9.35
Minimum	0.51	0.50	0.50
Maximum	34.95	35.44	35.28
Weibull scale	11.74	11.62	11.80
Weibull shape	1.82	1.71	1.87

Table 4.2: Descriptive statistics of actual and synthetic data

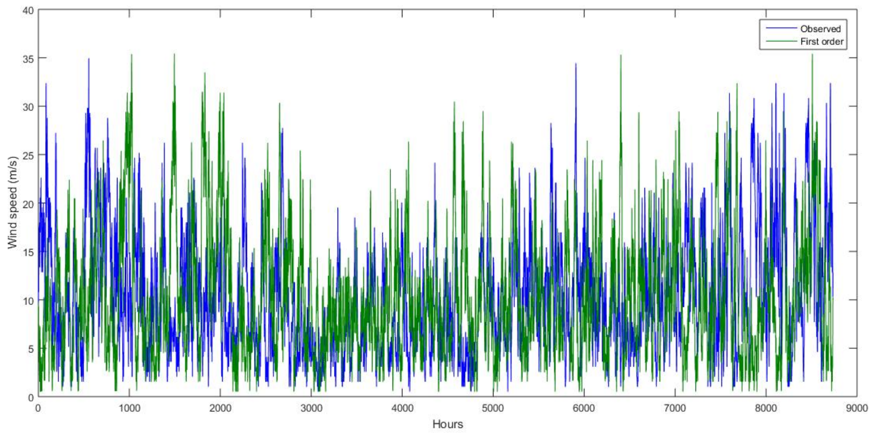

Figure 4.2: Synthetic against actual hourly wind speed of 2012 - first order

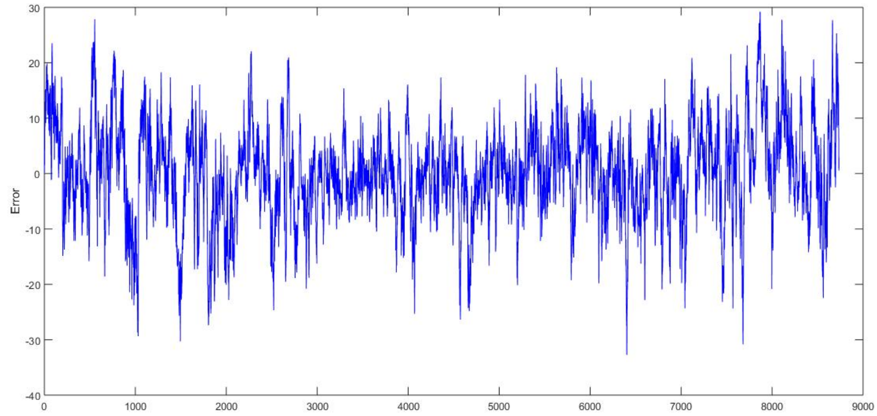

Figure 4.3: The simulation error of 1st order MC model

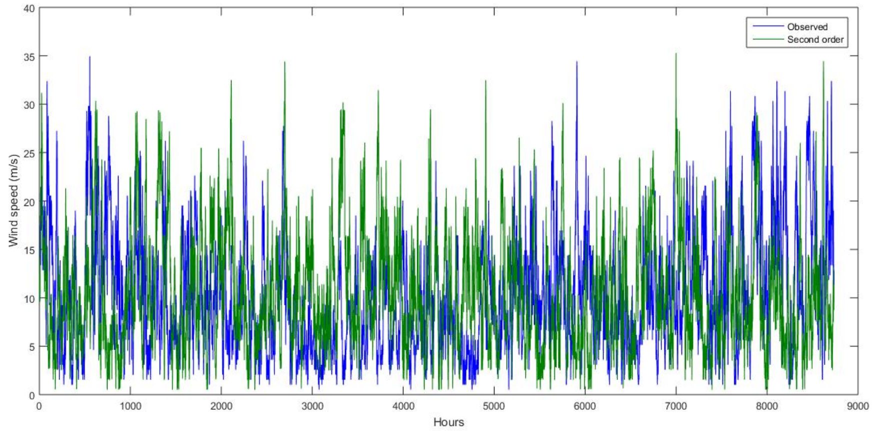

Figure 4.4: Synthetic against actual hourly wind speed of 2012 - second order

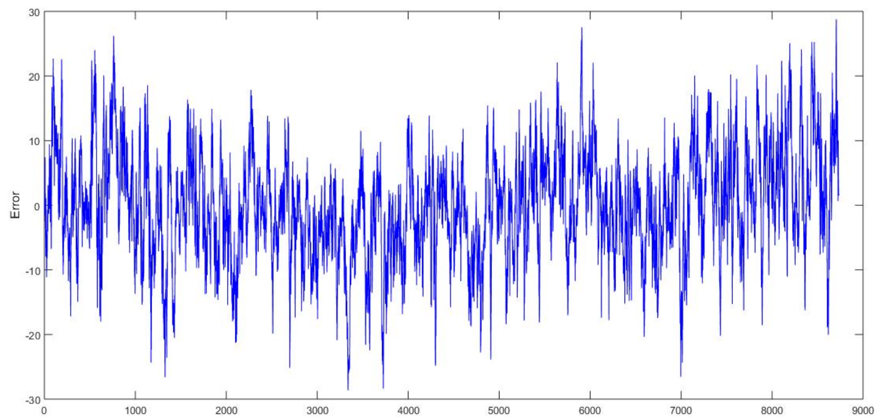

Figure 4.5: The simulation error of 2nd order MC model

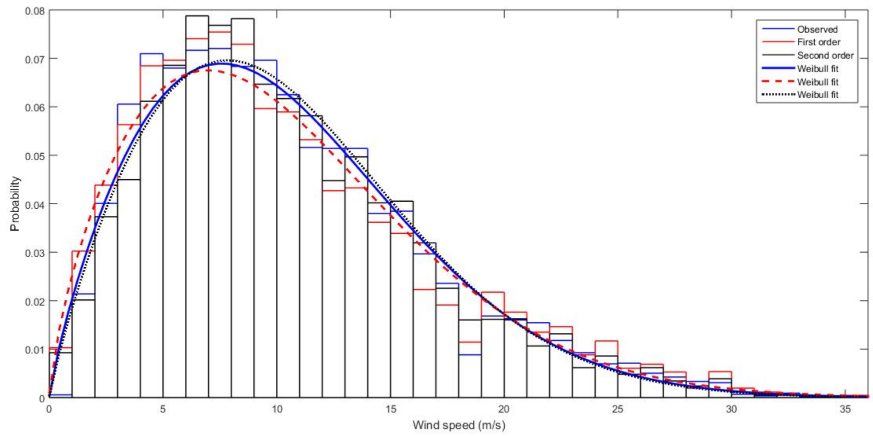

Figure 4.6: Probability distribution of actual and synthetic wind speed

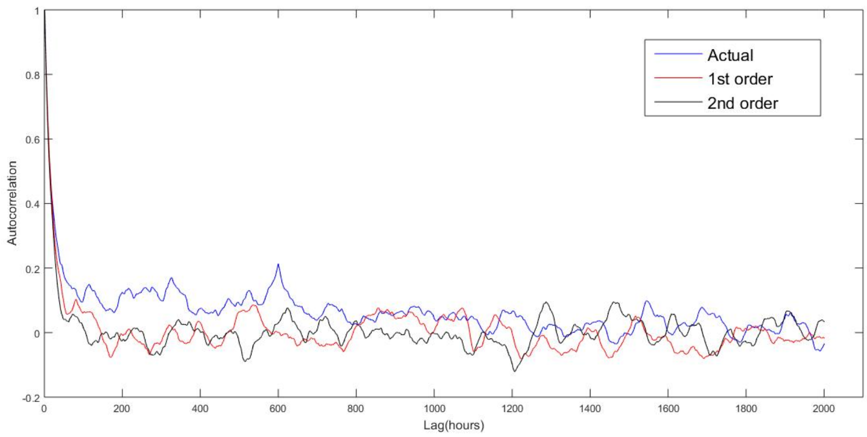

Figure 4.7: Autocorrelation functions of actual and synthetic wind speed

So far, MC models have been applied to the whole year dataset, information about seasonal variation (i.e. higher average wind speed in the winter, lower average wind speed in the summer) of wind speed may be lost. To introduce some seasonal variations in the wind output, I multiply the result from a non-seasonal MC model by a smooth function of time $F(t)$ that peaks in the winter and troughs in the summer. The fitted $F(t)$ is a function of sine and cosine with $R^2 = 0.99$. Monthly variation is shown in Figure 4.8 and the error plot is shown in Figure 4.9. The wind power is calculated for a wind turbine with rated power of 2 MW for actual and synthetic wind speeds (Figure 4.10), and the corresponding error plot is displayed in Figure 4.11. I calculate the signed relative error rather than unsigned, because it allows me to examine whether the wind speed is overestimated or underestimated.

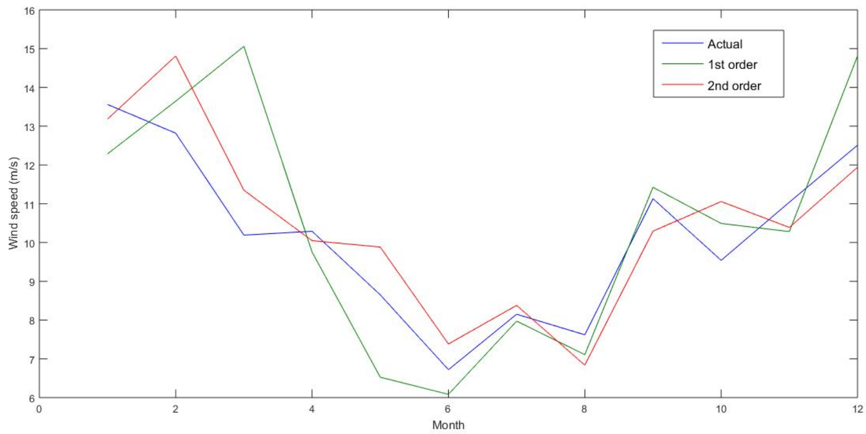

Figure 4.8: Synthetic against actual monthly wind speed of 2012

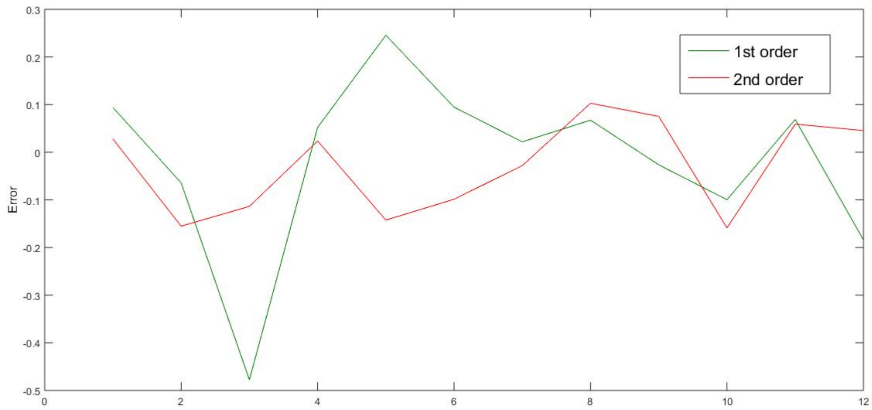

Figure 4.9: The signed relative error for wind speed of the 1st and 2nd order MC models

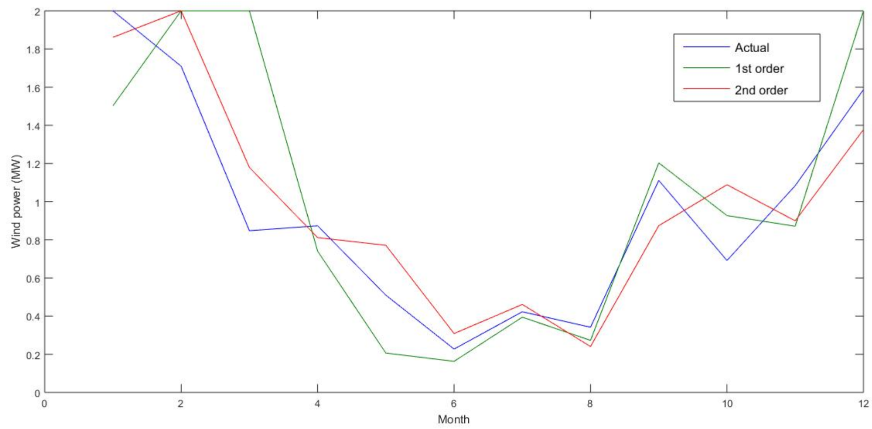

Figure 4.10: Synthetic against actual wind power of 2012

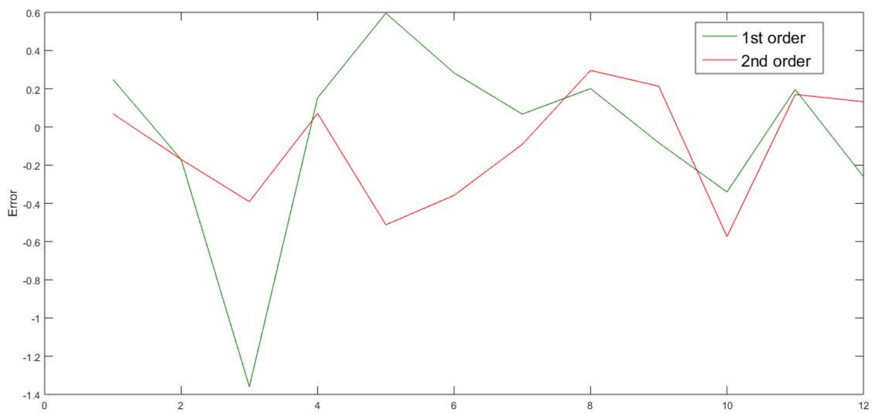

Figure 4.11: The signed relative error for wind power of the 1st and 2nd order MC models

4.1.2 Wind speed of Rugby

Similarly, I present the hourly wind speed of the year 2012 for Rugby in Figure 4.12. In Rugby, the wind is not as strong as in Aberdeen. The highest wind speed is only 11.31 m/s and the lowest speed is 0 m/s. With the presence of zero wind speed in the actual wind speed time series, I exclude the six data points with the value of zero when fitting the Weibull distribution. Wind speed is divided into 12 states with a difference of 1 m/s between the upper and lower limit (see Table 4.3). By examining the associated first order (12 $\times$ 12) and second transition probability matrices (144 $\times$ 12), similar findings have been revealed (refer to the description in Section 4.1.1), and the highest probability occurs on the diagonal.

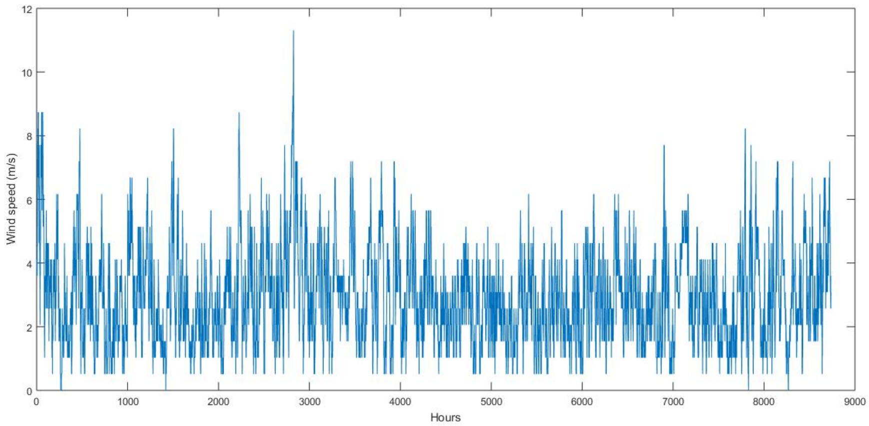

Figure 4.12: Hourly wind speed of year 2012

Wind speed (m/s)	State	Wind speed (m/s)	State
$0 \leq v \leq 1$	1	$6 < v \leq 7$	7
$1 < v \leq 2$	2	$7 < v \leq 8$	8
$2 < v \leq 3$	3	$8 < v \leq 9$	9
$3 < v \leq 4$	4	$9 < v \leq 10$	10
$4 < v \leq 5$	5	$10 < v \leq 11$	11
$5 < v \leq 6$	6	$11 < v \leq 12$	12

Table 4.3: No. of states of the wind speed data

Table 4.4 displays the general statistics and Weibull parameters of observed and simulated wind speeds. It is clear that both the first and second order MC models are sufficient to preserve most of the parameter values, but there is improvement in the statistical parameters of the second order MC model. Figure 4.13 displays the actual and synthetic data of a first order MC model and Figure 4.15 shows the synthetic and actual data of a second order MC model. The simulation error plots are displayed in Figure 4.14 and 4.16.

The probability distribution and the ACF plot of the observed and synthetic wind speed data are shown in Figure 4.17 and 4.18. It seems that the probability distribution of both observed and synthetic data are characterised by the Weibull distribution but the curve of the synthetic data by a second order MC model is closer to the curve of the observed data. From the ACF plot, it shows that the MC models produce some dependence structure of the time series. A second order MC model performs slightly better than a first order MC model, as the order of time lag increases.

	Actual	1st order MC	2nd order MC
Mean	2.940	3.086	3.055
Std deviation	1.488	1.705	1.547
Variance	2.214	2.908	2.394
Median	2.570	2.846	2.910
Minimum	0.000	0.000	0.000
Maximum	11.31	12.00	11.37
Weibull scale	3.321	3.440	3.413
Weibull shape	2.075	1.793	1.969

Table 4.4: Descriptive statistics of synthetic and actual data

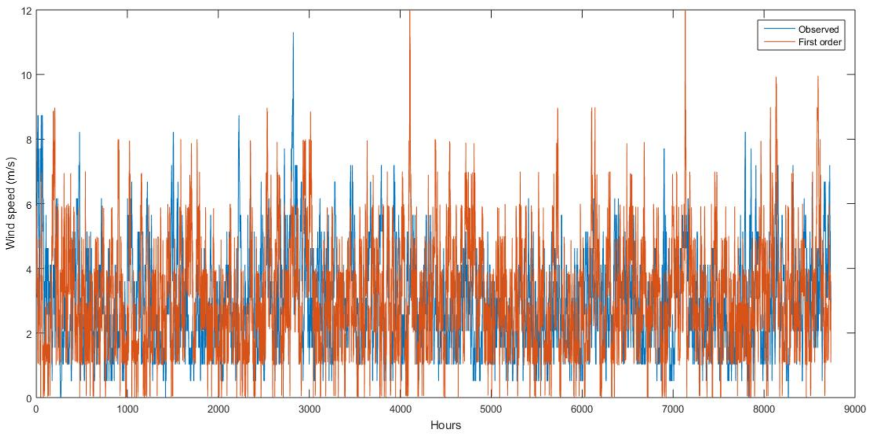

Figure 4.13: Synthetic against actual wind speed of 2012 - first order

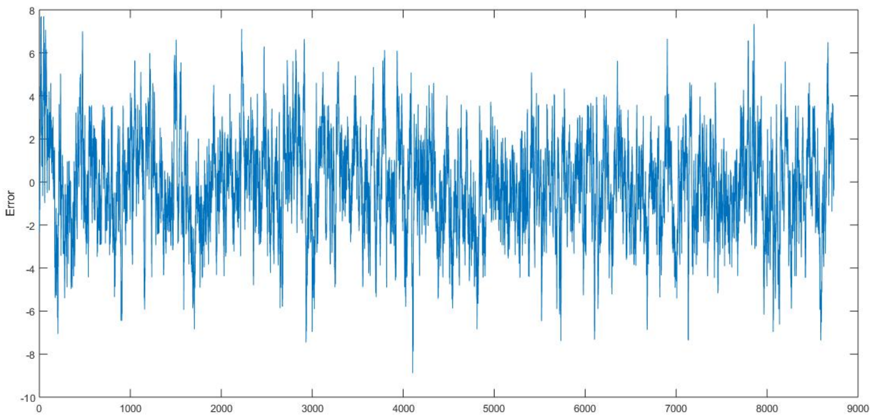

Figure 4.14: The simulation error of 1st order MC model

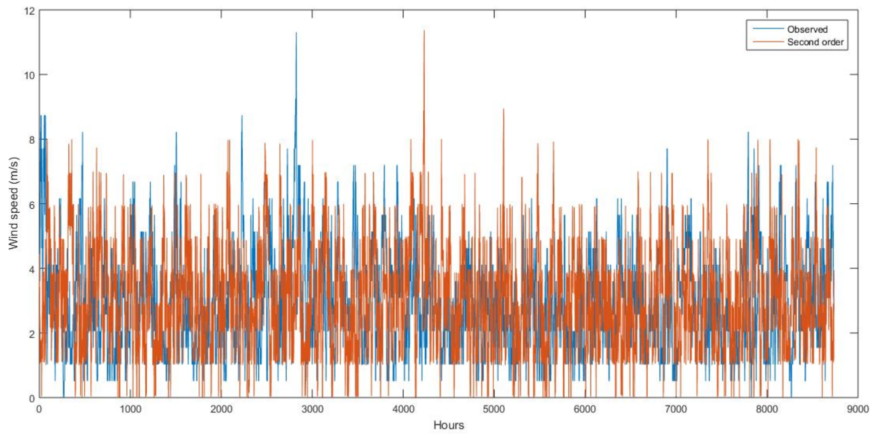

Figure 4.15: Synthetic against actual wind speed of 2012 - second order

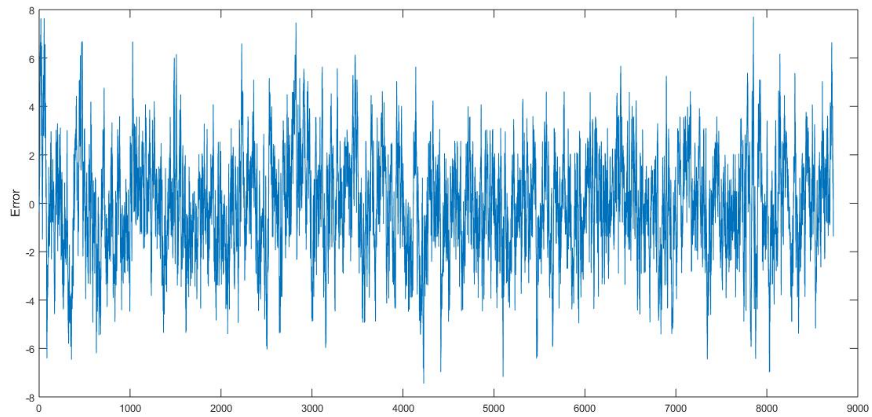

Figure 4.16: The simulation error of 2nd order MC model

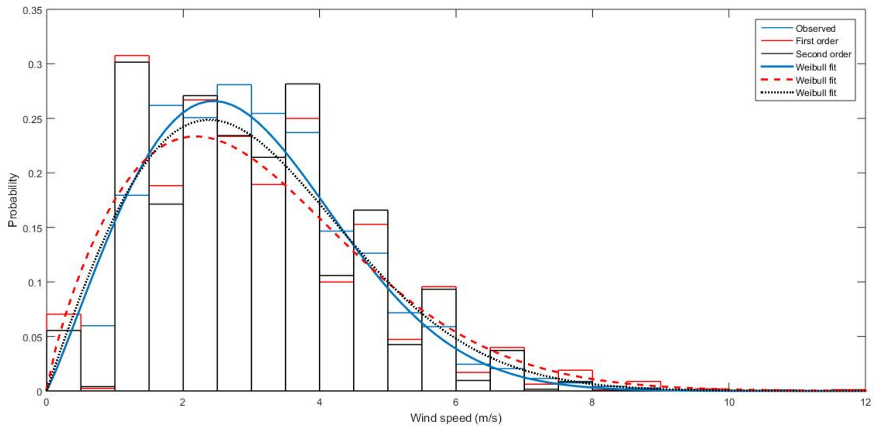

Figure 4.17: Probability distribution of observed and synthetic wind speed

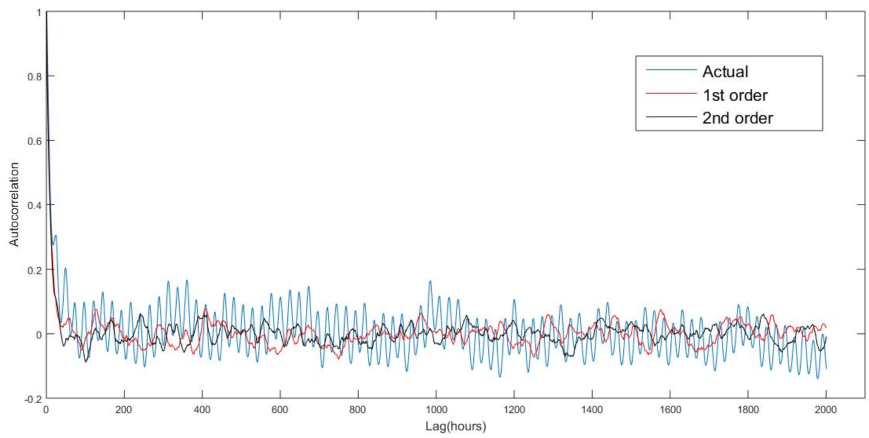

Figure 4.18: Autocorrelation functions of observed and synthetic wind speed

The obtained results from a non-seasonal MC model is multiplied by a smooth function of time $F(t)$ to introduce some seasonal variations in the wind output. The fitted $F(t)$ is a function of sine and cosine with $R^2 = 1$. Monthly variation is shown in Figure 4.19 and the signed relative error plot is shown in Figure 4.20. The wind power is also calculated for a wind turbine with rated power of 2 MW for actual and synthetic wind speeds (Figure 4.21), and the corresponding error plot is displayed in Figure 4.22.

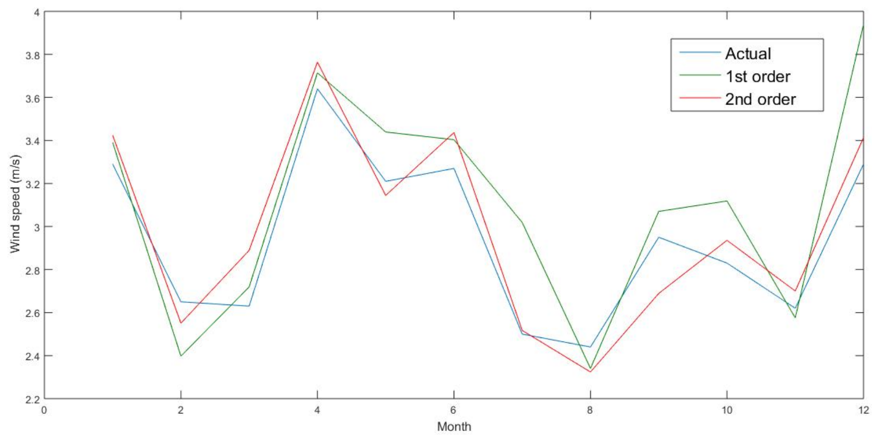

Figure 4.19: Synthetic against actual monthly wind speed of 2012

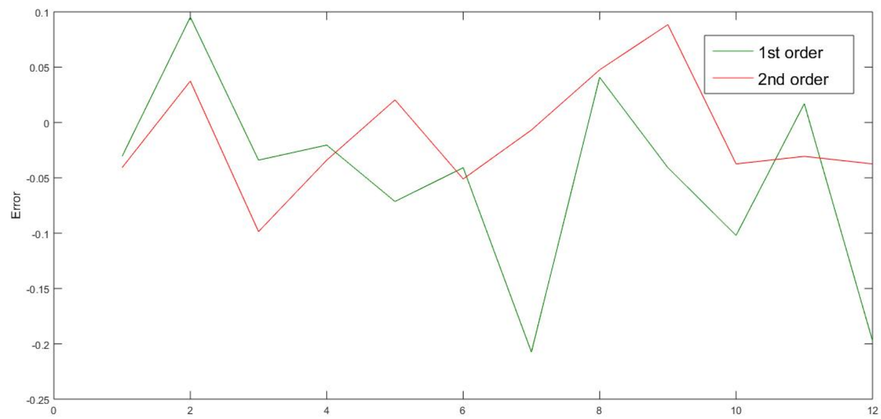

Figure 4.20: The signed relative error for wind speed of the 1st and 2nd order MC models

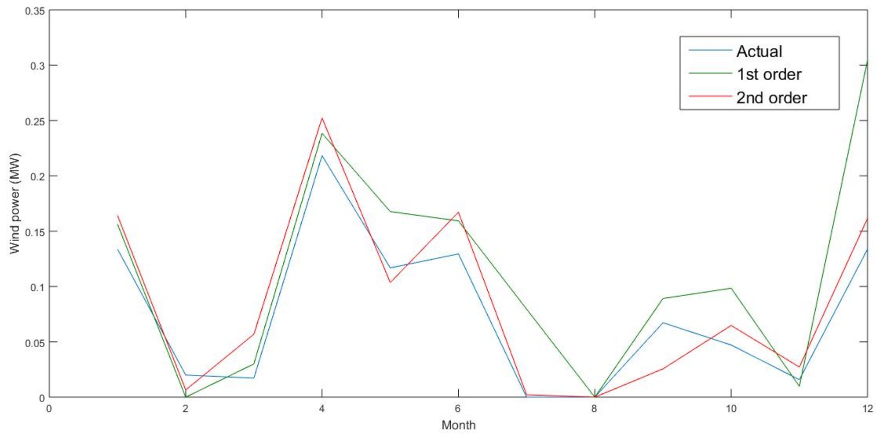

Figure 4.21: Synthetic against actual wind power of 2012

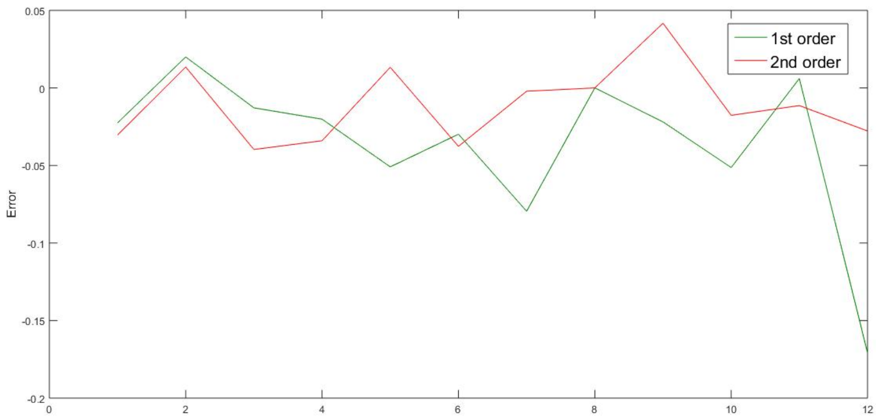

Figure 4.22: The signed error for wind power of the 1st and 2nd order MC models

4.1.3 General discussion

The comparisons between the actual and synthetic data reveal that the statistical characteristics are satisfactorily reproduced. The results suggest that a second order MC model generates relatively better results than a first order MC model.

Considering the variability of the simulation output is affected by the stochastic nature of the input wind speed data as well as by the random nature of the MC model itself, the MC simulation technique is sufficient to preserve most of the statistical characteristics and stochastic behaviour of wind speed time series, and it is possible to improve accuracy by using a second order MC model. A second order or higher MC model may be able to improve the results of synthetically generated wind speed data, but with the increase in the order, there is also an increase in the complexity of the model. Since no model is actually perfect, there will always be some hidden variables within the data that are not modelled. Thus, one can only hope for a model that can explain the data, but is not too complicated at the same time. The aim for this section is to show how successfully a MC model can be used to simulate wind speed data that capture the essential statistical properties of the observed wind speed data and account for the time correlation. It is found that with increasing lags in time, the success of this method in reproducing the correlation structure of the series decreases. Of course, a future topic would be to understand and try to find better ways of capturing information about the data.

4.2 Electricity demand

In addition to the intermittency of wind speed, electricity demand itself is also intermittent. To forecast the electricity demand, I use the method proposed by Filik et al. (2011). This method is selected due to its unique ability in long-term forecasting with an hourly accuracy. The selected method is able to make short-, medium- and long-term hourly load forecasting within a single framework. Traditionally, long-term forecasting is typically performed on a yearly average basis, whereas, hourly accuracy is used for short-term prediction. With this method, I am able to forecast the demand in the long-term with an hourly resolution.

The methodology is demonstrated with hourly actual loads from year 2010 to 2012 and annual energy consumption values from 2005 to 2012 for Aberdeen and Rugby. I apply this method to the whole dataset obtained, whereas, the original author excluded the data from public holidays. Annual sub-national consumption data are obtained from Department of Energy and Climate Change, and the half-hourly demand data are obtained from the National Grid website, followed by taking a sum to obtain hourly demand. Then, the data are scaled down to be representative at the sub-national level.

The method proposed by Filik et al. (2011) is a combination of three parts. The first step is to model the annual demand changes, then the modelling of the weekly residual variations within a year, and followed by the hourly variations within a week. The final model is constructed by combining these three parts. The steps will be explained in detail through the following two applications.

4.2.1 Electricity demand of Aberdeen

The hourly demand data from 2010 to 2012 for Aberdeen is shown in Figure 4.23, while Figure 4.24 displays the weekly demand load from 2010 to 2012. Both plots reveal the oscillatory behaviours of electricity demand. There seems to be a slight fall on the demand every year. The yearly demand load is presented in Figure 4.25 and it can be seen that the demand has a decreasing trend.

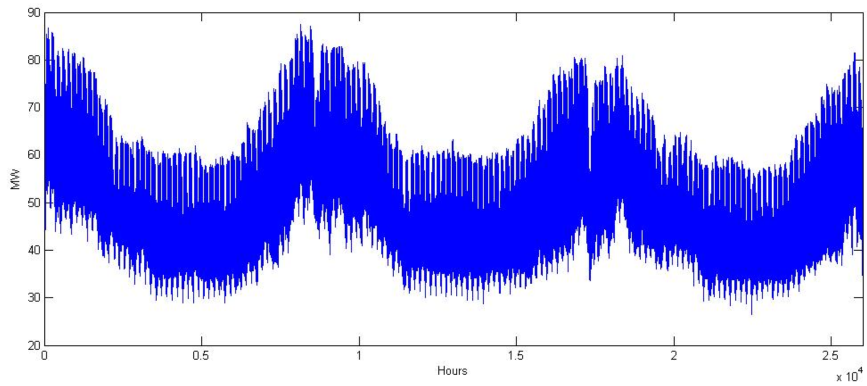

Figure 4.23: Hourly load data of year 2010 to 2012

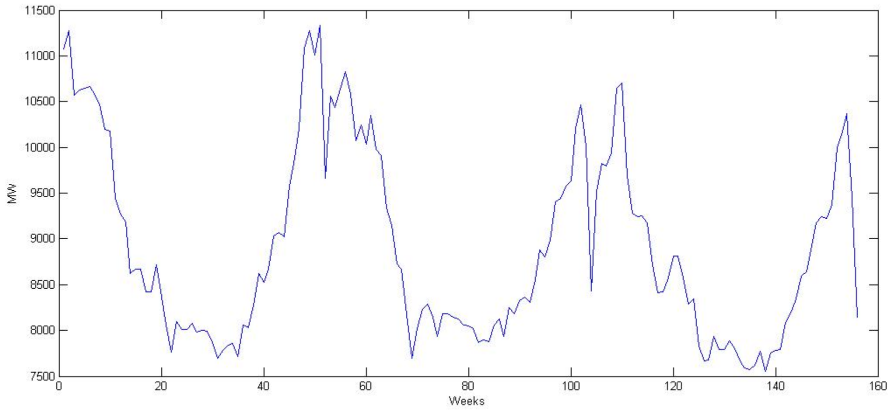

Figure 4.24: Weekly load data of year 2010 to 2012

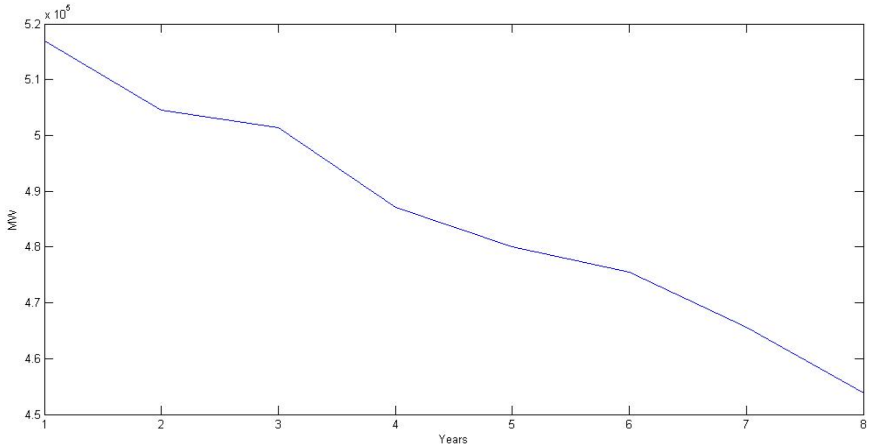

Figure 4.25: Yearly load data of year 2005 to 2012

I now go through the three steps of the methodology proposed by Filik et al. (2011), which include yearly modelling, weekly residual modelling and hourly residual modelling.

1. **Yearly trend modelling:** The first step is to express the demand in terms of weeks and estimate the change from year to year. I use the average weekly demand from 2005 to 2012 and fit a function that approximates the behaviour

on the yearly resolution. The yearly resolution can be approximated by various functions, such as linear, polynomial or power function. I consider a linear function with a goodness of fit R^2 of 0.988 and a cubic function with a goodness fit R^2 of 0.9903. In MATLAB, the default setting for confidence bounds is 95%.

Applying a curve fitting algorithm in MATLAB results in the following yearly load model (with 95% accuracy):

$$f_1' = -3.175x + 10080, \quad x = 52, \ldots, 416,$$

or

$$f_2' = -1.506 \times 10^{-5} x^3 + 0.01053x^2 - 5.254x + 10190, \quad x = 52, \ldots, 416,$$

where $x = 52$ denotes the last week of 2005, and $x = 416$ denotes the last week of 2012. A comparison between f_1', f_2' and the demand data are shown in Figure 4.26 and 4.27, respectively.

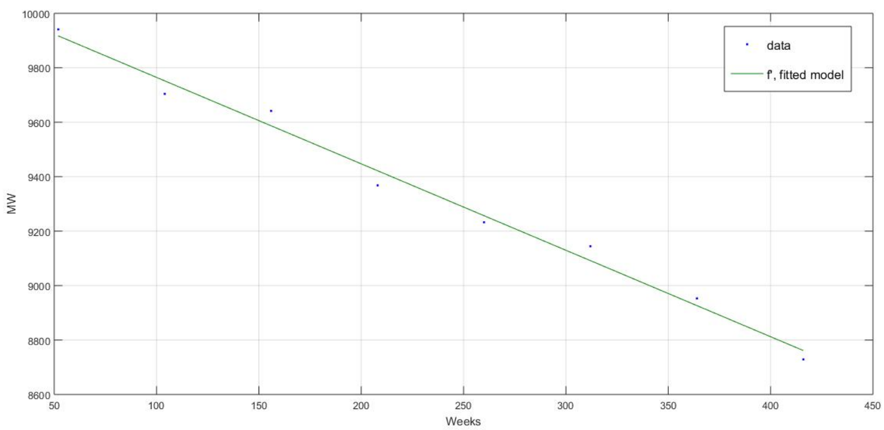

Figure 4.26: Model approximation of yearly trend data-linear f_1'

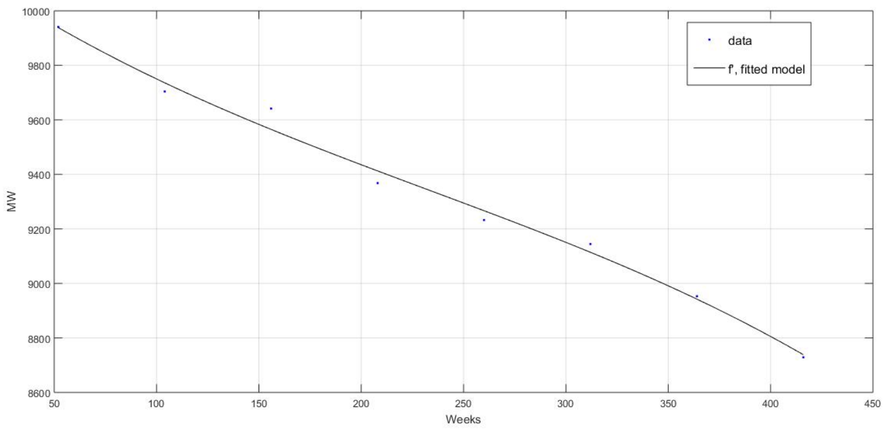

Figure 4.27: Model approximation of yearly data-cubic f_2'

2. **Weekly residual modelling:** I assume that the weekly load model is a combination of the yearly trend model and variations on a weekly basis. The weekly variation is obtained by dividing the yearly averaged data, which I name the weekly residual load variations within a year. Since I only have hourly data from 2010 to 2012, I compute the weekly residual data for the period 2010 to 2012 and consider fitting a model $g(x)$. Here, I have chosen to model $g(x)$ by a sum of three sine functions due to the relatively oscillatory behaviour of the available data (see Figure 4.24). The chosen function is considered to be a sum of three terms, because the goodness of fit R^2 does not increase significantly with increasing number of terms. For instance, increasing the number of terms from three to eight results in an increase of R^2 from 0.8375 to 0.9172. There are other functions that fit the data well, for example, using fourier series I have R^2 = 0.8363 for a three terms sum and R^2 = 0.9119 for an eight terms sum. Applying a curve fitting algorithm in MATLAB and the

resulting functional form (with 95% accuracy) for g is:

$$g_1(x) = 3.227\sin(0.004007x + 7.472) + 1.97\sin(0.006008x + 10.11)$$
$$+ 0.1459\sin(0.1198x + 1.657),$$
or
$$g_2(x) = 1.778\sin(0.02474x - 0.4493) + 1.048\sin(0.04735x - 4.898)$$
$$+ 0.3792\sin(0.1236x + 0.7138) + 0.381\sin(0.08052x - 0.2783)$$
$$+ 0.03677\sin(0.2369x + 2.924) + 0.6434\sin(0.1546x - 0.03188)$$
$$+ 0.5148\sin(0.16x + 13.93) + 0.01608\sin(0.6112x + 1.106),$$
or
$$g_3(x) = 0.9974 + 0.1463\cos(0.1199x) - 0.004263\sin(0.1199x),$$

where x starts in week 261 from 2005, i.e., week 1 of 2010. A comparison between g_1, g_2, g_3 and the actual data are shown in Figures 4.28, 4.29 and 4.30, respectively.

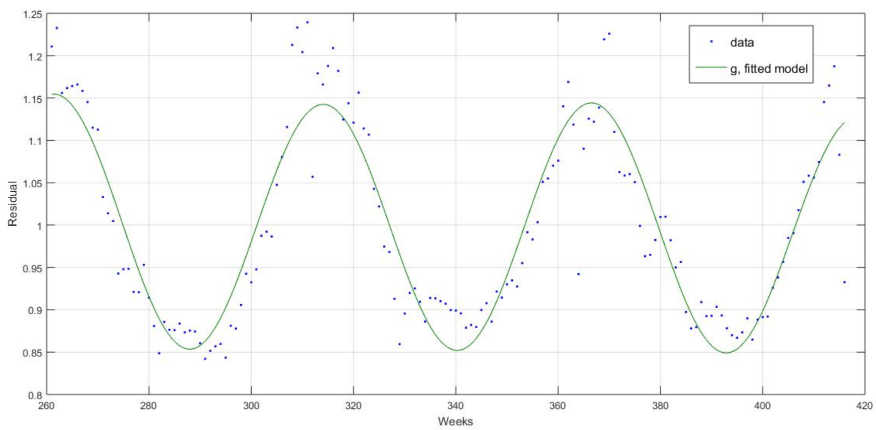

Figure 4.28: Model approximation of weekly residual data-sine of 3 terms g_1

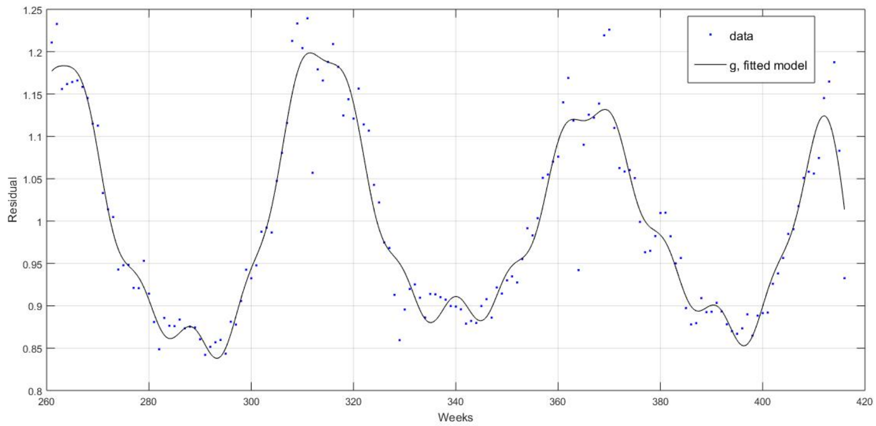

Figure 4.29: Model approximation of weekly residual data-sine of 8 terms g_2

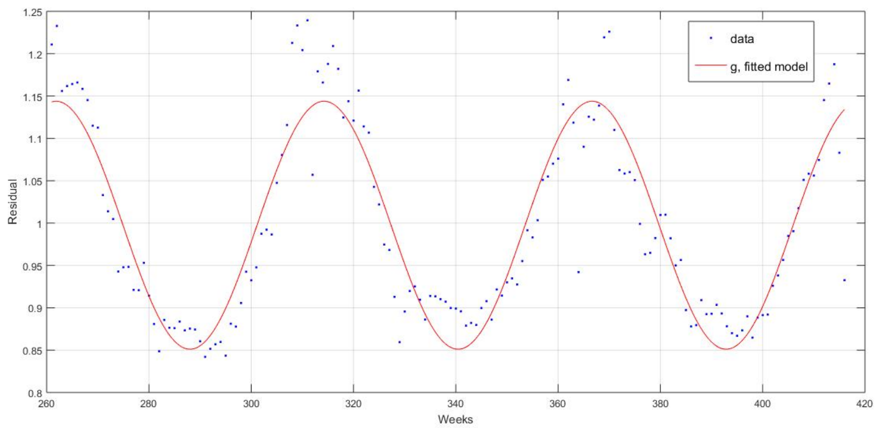

Figure 4.30: Model approximation of weekly residual data-fourier series g_3

I assume that the weekly residual load variation for 2005 to 2010 can be modelled similarly to the weekly residual load variation for 2010 to 2012. In other words, I can extend the domain of g_1 to the period 2005 to 2012. I define the demand function with weekly resolution, $D'(x)$, as the product f_1' and g_1:

$$D'(x) = f_1'(x) \cdot g_1(x), \quad \text{for } x = 1, \ldots, 416.$$

3. **Hourly modelling:** I assume that the hourly load model is a combination of the weekly load model and variations on an hourly basis. I observe that the hourly variations within a single week are quite similar throughout the year, so I assume that the hourly variations within a week can be well represented by the averaged hourly load shape across a week. I denote this shape as the week-to-hour template, and the template for year 2011 can be found in Figure 4.31. Since I have three years period of hourly data, to avoid the effects of overall increase or decrease within a year, I normalise the template so that the volume under the surface is 1 (see Figure 4.32).

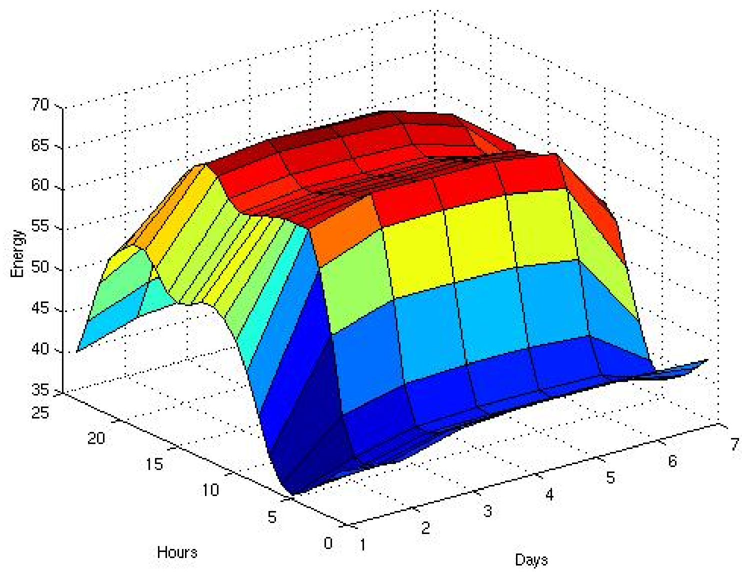

Figure 4.31: Averaged hourly variations visualised in 2-D within a week

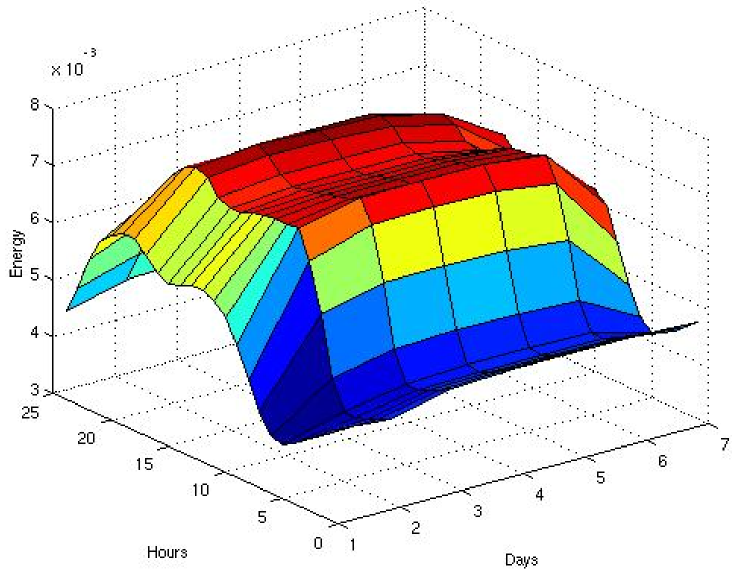

Figure 4.32: Normalised hourly variations visualised in 2-D within a week

I denote the week-to-hour template by $T(h,d)$, where d represents the days of the week and h represents the hours of the day d. To obtain $T(h,d)$, I consider the week-to-hour template as a 24×7 matrix, and apply the 2-D discrete cosine transform (2-D DCT). Next, I pick out the seven entries with the largest magnitude and set all other entries to zero. I then apply the inverse 2-D discrete cosine transform to obtain $T(h,d)$. In this case, only seven coefficients are required to generate the week template data. The week-to-hour template is shown in Figure 4.33.

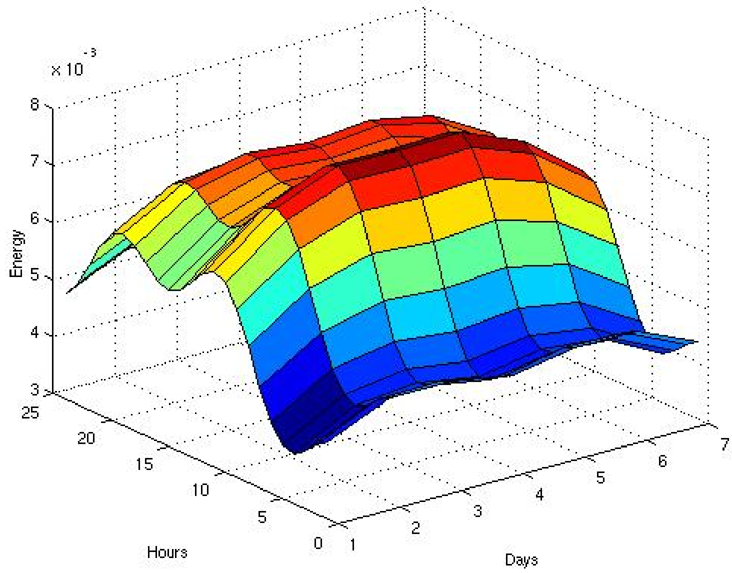

Figure 4.33: The week template and modelling surface

I now assume that the week-to-hour template is representative of the hourly variation within a week. Thus, the demand model with hourly resolution is

$$D(h,d,x,y) = D'(\hat{x}) \cdot T(h,d),$$

for $h = 1,2,\ldots,24$; $d = 1,2,\ldots,7$; $x = 1,2,\ldots,52$ and $y = 1,2,\ldots,8$.

where

$$D'(\hat{x}) = f_1'(\hat{x}) \cdot g_1(\hat{x}), \quad \hat{x} = (52y+x),$$

while h,d,x and y indicate respectively hours, days, weeks and years from 2005 to 2012.

Finally, the demand model established in the above three-step procedure is validated by comparing the predicted values with the actual values for year 2012, which is shown in Figures 4.34, 4.35 and 4.36. The signed relative error is calculated and displayed in Figure 4.37.

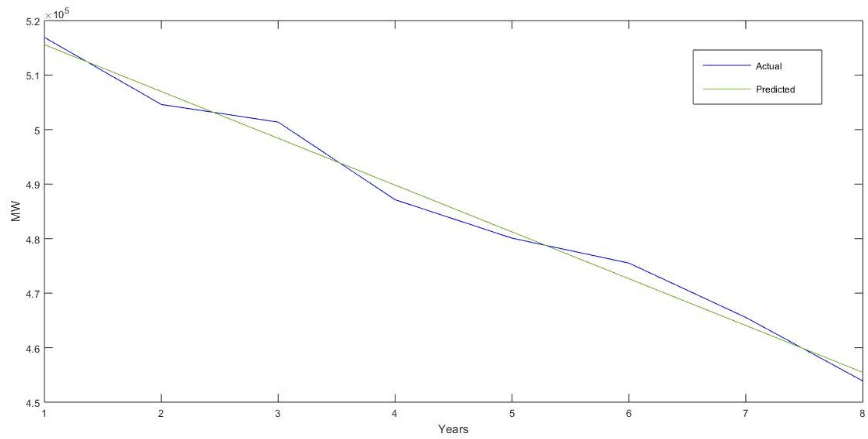

Figure 4.34: Predicted against actual demand of 2005 to 2012 - yearly

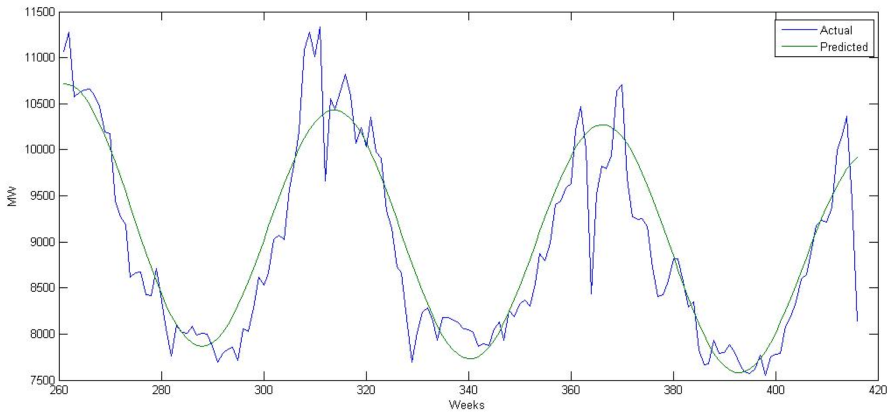

Figure 4.35: Predicted against actual demand of 2010 to 2012 - weekly

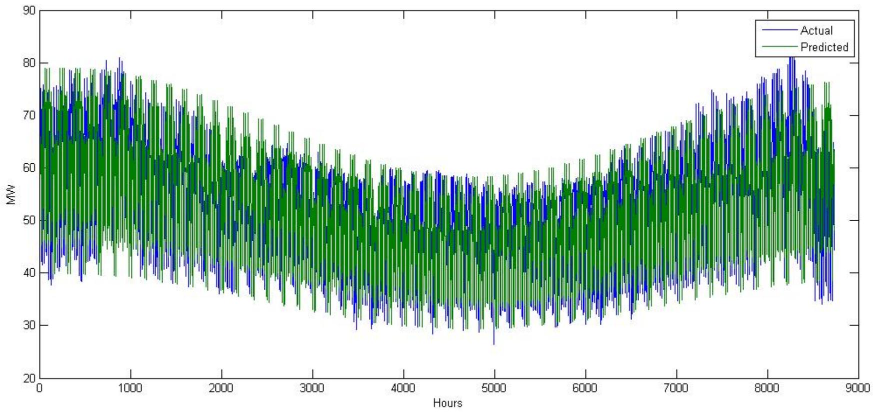

Figure 4.36: Predicted against actual demand of 2012 - hourly

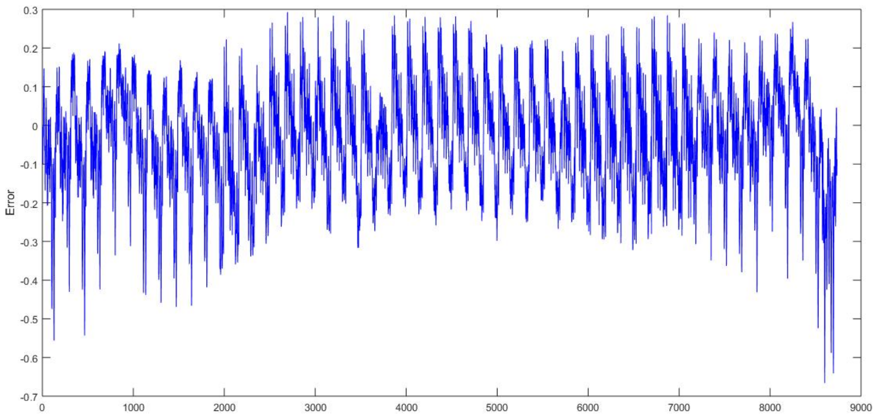

Figure 4.37: Signed relative error of $D(h,d,x,y)$ of 2012 - hourly

4.2.2 Electricity demand of Rugby

Similarly, I show the hourly demand data from 2010 to 2012 for Rugby in Figure 4.38, and Figure 4.39 represents the weekly demand from 2010 to 2012. The electricity demand is lower in Rugby, but there are also indications of oscillatory behaviours. The annual hourly demand is plotted in Figure 4.40 and it shows a decreasing trend.

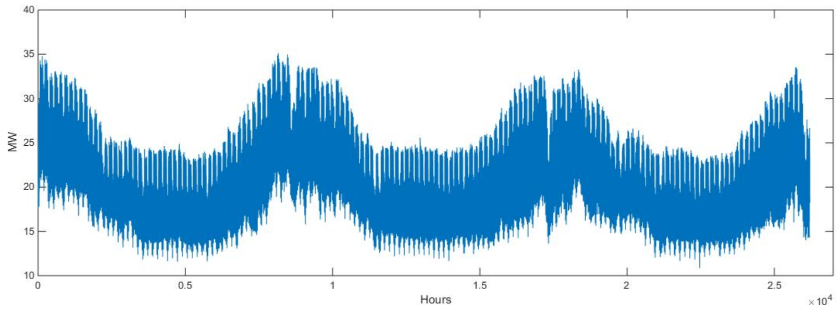

Figure 4.38: Hourly load data of year 2010 to 2012

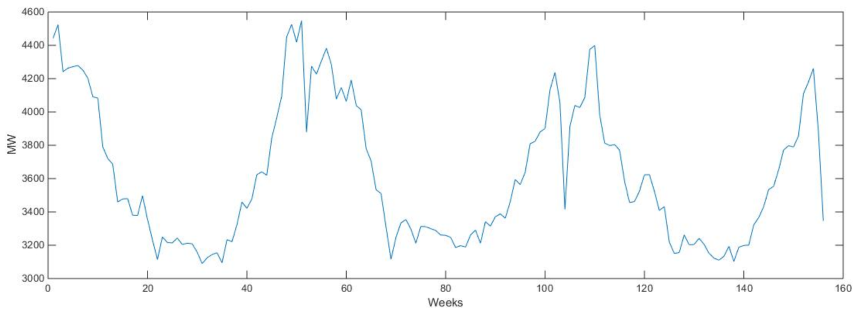

Figure 4.39: Weekly load data of year 2010 to 2012

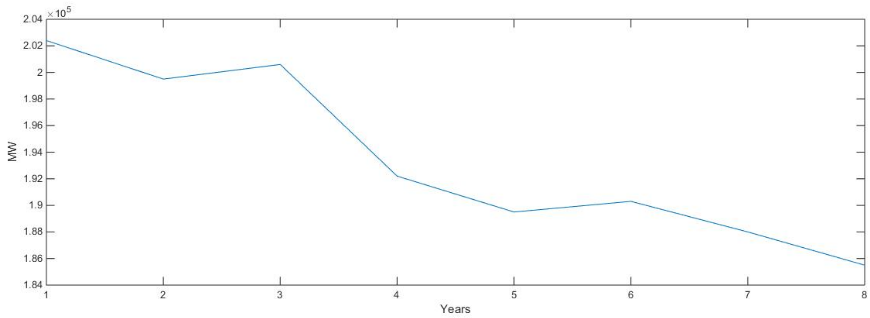

Figure 4.40: Yearly load data of year 2005 to 2012

1. **Yearly trend modelling:** I use the demand from 2005 to 2012 and fit a linear function which approximates the behaviour on the yearly resolution in MATLAB. The resulting yearly load model by applying a curve fitting algorithm

in MATLAB (with 95% accuracy) is:

$$f'' = -0.9219x + 3937, \quad x = 52, \ldots, 416,$$

where $x = 52$ denotes the last week of 2005 and $x = 416$ denotes the last week of 2012. A comparison between f'' and the yearly load data is shown in Figure 4.41.

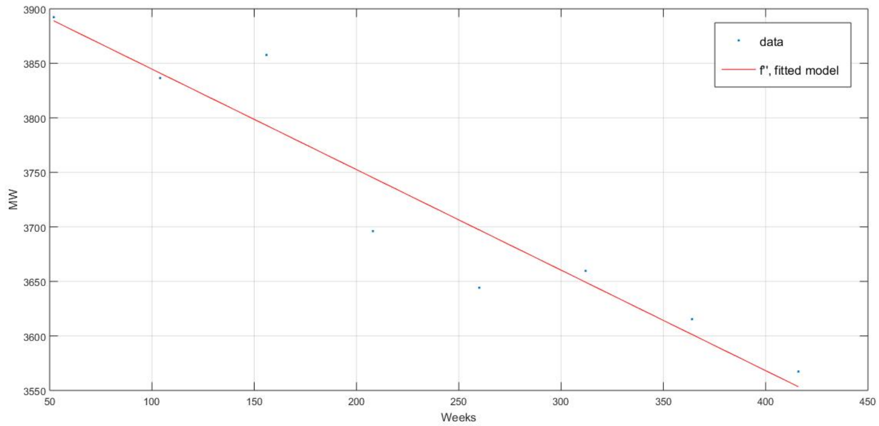

Figure 4.41: Model approximation of yearly data

2. **Weekly residual modelling:** I assume that the weekly load model is a combination of the yearly load model and variations on a weekly basis. To obtain the weekly variation, I divide by the yearly averaged data. This is the weekly residual load variations within a year. I then compute the weekly residual data for the period 2010 to 2012 and consider fitting a model $g'(x)$. Again, I have chosen $g'(x)$ to be a sum of three sine functions for similar reasons described in the Aberdeen application. I apply a curve fitting algorithm in MATLAB and the resulting functional form (with 95% accuracy) for g' is:

$$g'(x) = 2.871\sin(0.004309x + 7.268) + 1.706\sin(0.006457x + 9.862)$$
$$+ 0.1464\sin(0.1197x + 1.669),$$

where x starts in week 261 from 2005, i.e., week 1 of 2010. A comparison between g' and the actual data is shown in Figure 4.42.

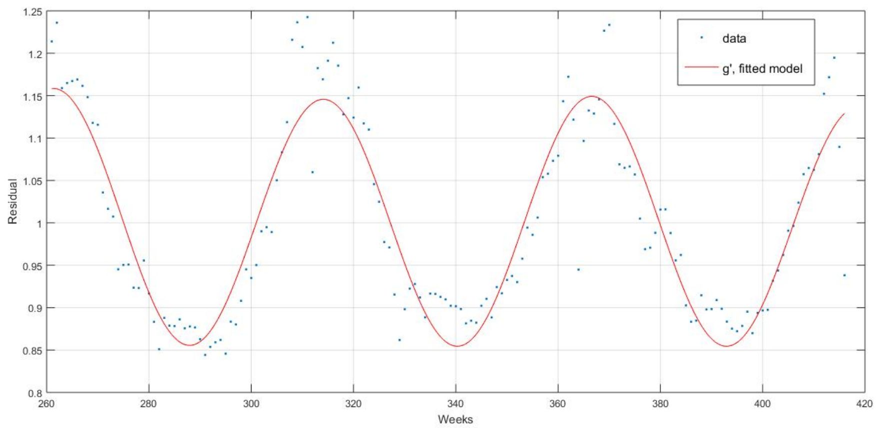

Figure 4.42: Model approximation of weekly residual data

I additionally assume that the weekly residual load variation for 2005 to 2010 can be modelled similarly to the weekly residual load variation for 2010 to 2012. In other words, I extend the domain of g' to the period 2005 to 2012. I define the demand load function with weekly resolution, $D''(x)$, as the product of f'' and g':

$$D''(x) = f''(x) \cdot g'(x), \quad \text{for } x = 1, \ldots, 416.$$

3. **Hourly modelling:** Similarly to the above, I assume that the hourly load model is a combination of the weekly load model and variations on a hourly basis. Figure 4.43 displays the week-to-hour template for year 2011, and Figure 4.44 shows the normalised template. Let $T'(h,d)$ denote the week-to-hour template, where d represents the days of the week and h represents the hours of the day d, (see Figure 4.45 for the modelling surface).

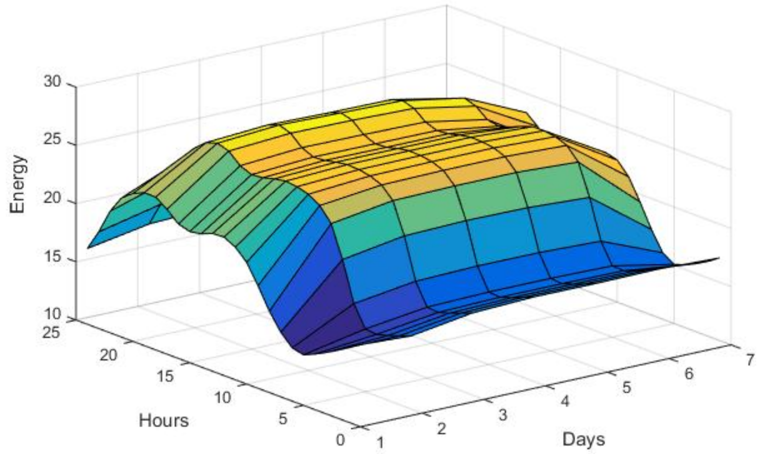

Figure 4.43: Averaged hourly variations visualised in 2-D within a week

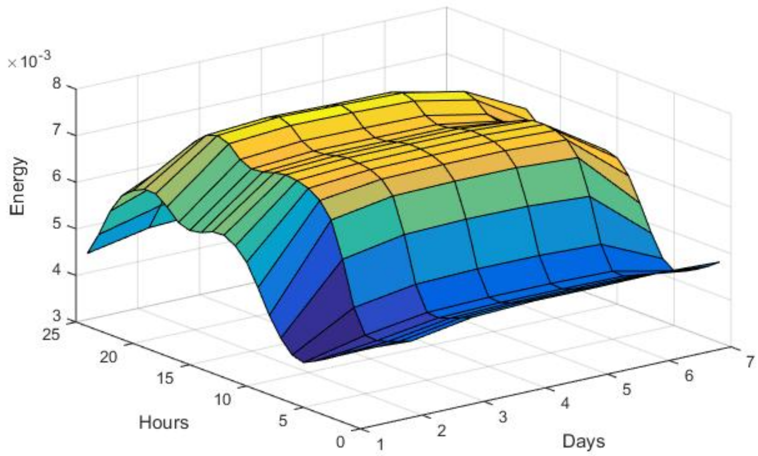

Figure 4.44: Normalised hourly variations visualised in 2-D within a week

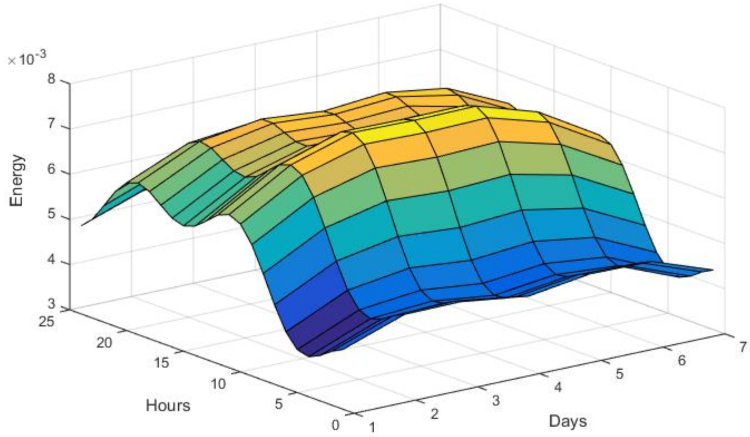

Figure 4.45: The week template and modelling surface

Then, the demand model with hourly resolution is

$$D(h,d,x,y) = D''(\hat{x}) \cdot T'(h,d),$$

for $h = 1,2,\ldots,24$; $d = 1,2,\ldots,7$; $x = 1,2,\ldots,52$ and $y = 1,2,\ldots,8$.

where

$$D''(\hat{x}) = f''(\hat{x}) \cdot g'(\hat{x}), \quad \hat{x} = (52y + x),$$

while h,d,x and y indicate respectively hours, days, weeks and years from 2005 to 2012.

Finally, the demand model is validated by historical data. I compare the predicted value with the actual value for the year 2012, which is shown in Figures 4.46, 4.47 and 4.48. Figures 4.49 displays the error terms. With a relatively short periods of data, the predicted values match the actual values reasonably well.

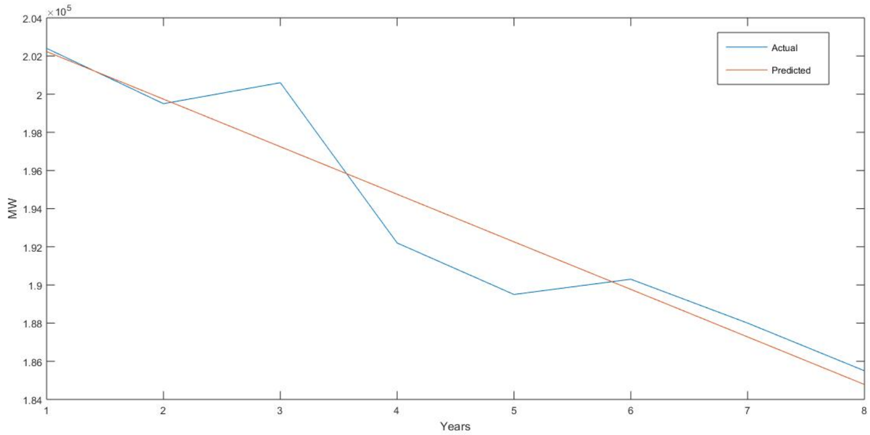

Figure 4.46: Predicted against actual demand of 2005 to 2012 - yearly

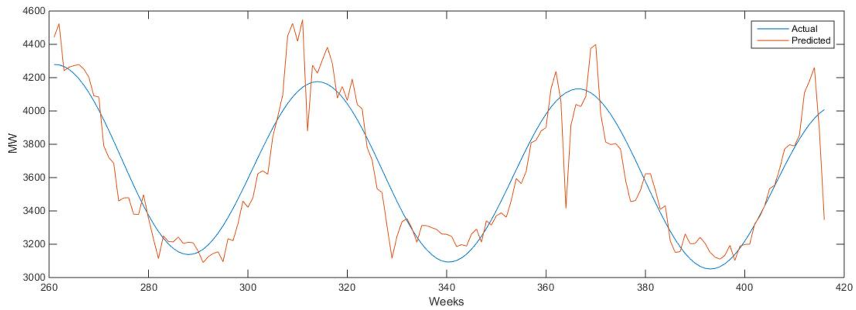

Figure 4.47: Predicted against actual demand of 2010 to 2012 - weekly

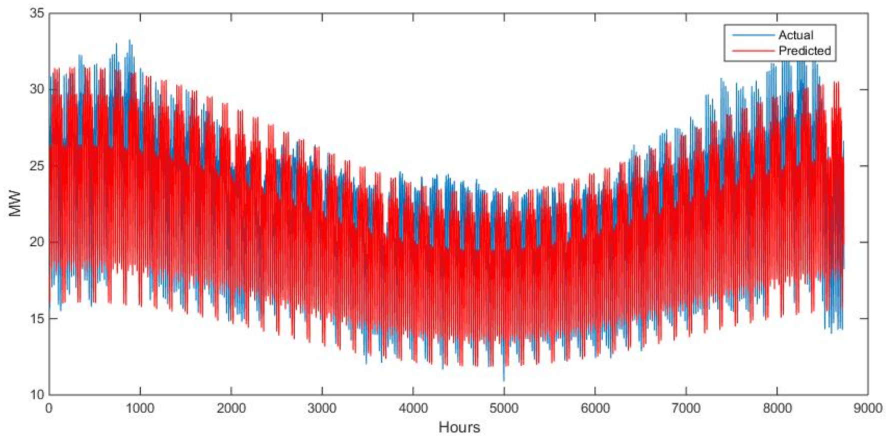

Figure 4.48: Predicted against actual demand of 2012 - hourly

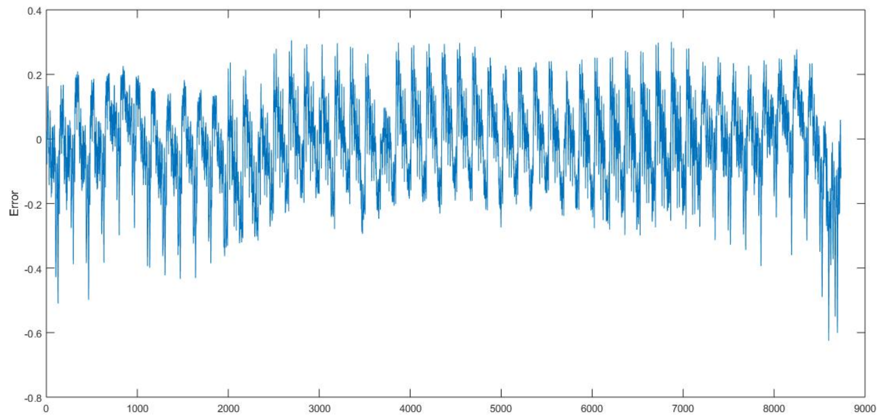

Figure 4.49: Signed relative error of $D(h,d,x,y)$ of 2012 - hourly

4.2.3 General discussion

From the above analysis, the demand model works quite well for both Aberdeen and Rugby. The reason to calculate the signed relative error is that it would be useful to see when and where the negative values occur. In practice, those negative values are more costly than positive values. When the predicted values are smaller than actual values, this causes blackouts. The error terms are higher at the beginning and the end of the plot, this could be due to the holiday effect (i.e. Christmas and New Year).

In the modelling of hourly variations, I have made the assumption that hourly variations within a week of the year can be represented by the averaged hourly load shape across a week. This would smooth out the hour resolution, and thus the predicted demand pattern tends to be flatter than its actual pattern. Due to the inaccessibility and unavailability of actual hourly electricity demand data at a regional level, the data used for testing are scaled down from a national level. Actual regional hourly consumption pattern could differ from the national consumption patterns as the consumption behaviour varies from region to region. In addition, for the yearly modelling, I have the limitation that only a few years of data are available to test the model. From the eight data points, I can deduce only a decreasing trend. However, it is not clear whether it is a linear relationship. If a much longer period of data were present, I could fit a model with more accuracy and predict more accurately. For the weekly and hourly modelling, it may be helpful to have a longer period of data for further observation and finer tuning. It would certainly help increase the goodness of fit R^2 in the weekly and hourly fitting. However, I do not expect these changes to be significant.

4.3 Numerical example

I provide numerical examples as I have done in Section 3.3, but, rather than using real data, I use the simulated wind speed from a second order MC model and the predicted electricity demand from the model described in Section 4.2. Meanwhile, I keep the other parameters the same as in Section 3.3. The numerical examples presented in this section are contained in Appendix B.2.

4.3.1 Results and discussion

Let $(\overline{Y_r}^*, \overline{W_r}^*, \overline{S_r}^*)$ denote the vector of optimal solutions.

Type	Aberdeen	Rugby
Gas-fired plant $\overline{Y_r}^*$	12	11.93
Wind turbine $\overline{W_r}^*$	15.24	10
Battery $\overline{S_r}^*$	30	30

Table 4.5: Optimal capacities for energy storage and generators

With the same choice of parameter values, the optimisation algorithm returns an optimal capacity $(\overline{Y_r}^*, \overline{W_r}^*, \overline{S_r}^*) = (12, 15.24, 30)$ for Aberdeen and $(\overline{Y_r}^*, \overline{W_r}^*, \overline{S_r}^*) = (11.93, 10, 30)$ for Rugby. The corresponding total costs over the year 2012 are £7.00 million and £6.56 million, respectively.

The optimal capacities have some difference with the optimal capacities obtained in Section 3.3. This is within expectation as both wind speed and demand data have some differences to the actual data.

Chapter 5

Model extension

To increase the flexibility of the developed model framework and improve the efficiency of the system, I consider three model extensions. I first study the effect of other storage technologies, as different types of energy storages have different characteristics including round-trip efficiency, duration, investment and O/M cost and lifetime, I modify the part of the cost functional, the round-trip efficiency and the maximum charging/discharging capacity corresponding to the storage. Secondly, I include the carbon emission modelling from the gas-fired plant by applying a carbon tax or carbon emission cap. The third extension is to consider the possibility of connecting the system to the National Grid, where I import from the National Grid when the system has an energy shortage, and I export the surplus to the National Grid provided that the electricity demand is met and the storage is fully filled. By considering this interconnected network, I would be able to make better use of the renewable generation. I consider two scenarios, one with the gas-fired plant and one without the gas-fired plant. All the numerical examples presented in this chapter are included in Appendix B.3.

5.1 Notation

A_c: Fixed component of distribution cost

A_d: Variable component of distribution cost

G_t: Electricity imported from the National Grid in hour t (MWh)

H_t: Electricity exported the National Grid in time period t (MWh)

P_e, P_s: Price of importing and exporting

P_{co_2}: Carbon price

E_{co_2}: Actual carbon emission

E_0: Carbon emission baseline

ζ: Carbon emission factor

5.2 Different types of storage technologies

So far, I have considered only battery energy storage in the numerical examples, in fact, there is a broad variety of storage technologies available as described in the introduction. Different types of storages have different cost, lifetime and round-trip efficiency, etc. Therefore, I would like to compare how the optimal solution varies according to the different types of storage. In this section, I do not look into the social benefit side.

5.2.1 Numerical example

Information on the different types of energy storage is taken from (Gönen, 2011; Komor and Glassmire, 2012; Battke et al., 2013; Carnegie et al., 2013; Akhil et al., 2013; Luo et al., 2015). The investment and O/M cost, efficiency rate, duration and lifetime are given in the Table 5.1. Here, BES denotes battery energy storage, CAES denotes compressed air energy storage, FES denotes flywheel energy storage and PHS denotes pumped hydroelectric storage. The investment cost presented is in the unit of million pounds per megawatt hour (M£/MWh) and the O/M cost is given as pounds per megawatt hour (£/MWh). The lifetime is measured in years and the duration is measured in hours.

Type	Investment cost (M£/MWh)	Lifetime (years)	O/M cost (£/MWh)	Round-trip efficiency (%)	Duration (hours)
BES	2.3	15	3.6	90	1
FES	6.2	15	3.1	85	1
PHS	0.3	40	3.1	81	10
CAES	0.1	30	2.3	70	10

Table 5.1: Information the different types of energy storage technologies

Looking at the investment cost, FES has the highest investment cost, whereas PHS and CAES are much cheaper. BES and FES can be charged or discharged very quickly, CAES and PHS take much longer to get charged or discharged. Therefore, the amount of energy that charged into or discharged from the energy storage is more limited for PHS and CAES. That is, if 10 units of surplus energy are generated, BES and FES could take all in, but only 1 unit can be charged for PHS and CAES. The round-trip efficiency is highest for BES and CAES has the lowest efficiency rate among the four types of storage technologies. The typical lifetime is longer for PHS and CAES.

5.2.1.1 Results and discussion

Let $(\overline{Y_r}^*, \overline{W_r}^*, \overline{S_r}^*)$ denote the vector of optimal solutions.

Type	BES	FES	PHS	CAES
Gas-fired plant $\overline{Y_r}^*$	12	12	12	11.84
Wind turbine $\overline{W_r}^*$	15.24	18.36	11.91	11.65
Storage $\overline{S_r}^*$	30	30	35	40
Total cost (M£)	7.00	15.05	2.41	2.23

Table 5.2: Optimal capacities for energy storage and generators under different storage technologies - Aberdeen

Type	BES	FES	PHS	CAES
Gas-fired plant $\overline{Y_r}^*$	11.93	12	12	12
Wind turbine $\overline{W_r}^*$	10	10.01	10.03	11.07
Storage $\overline{S_r}^*$	30	30	30	30
Total cost (M£)	6.56	14.37	2.20	2.16

Table 5.3: Optimal capacities for energy storage and generators under different storage technologies - Rugby

Re-running the model, it can be seen that optimal capacities have changed across the different storage technologies. Aberdeen has a more volatile wind speed, it can generate a lot of surplus energy. With a lower charging capacity for CAES and PHS, it is suggested to have a bigger capacity in order to make better use of the surplus. Meanwhile, the cost for CAES and PHS is relatively lower, with sufficient surplus, it is more cost-effective to invest in the storage than in wind turbine and gas plant. On the other hand, flywheels are very expensive and the efficiency is slightly lower compared to batteries, it will need a greater supply from the wind turbine. In Rugby, wind condition is more stable, the surplus generated is smaller in comparison to Aberdeen. After meeting its electricity demand, there is little left to be charged. When the storage has lower efficiency rate, it will just suggest a slightly bigger wind turbine to overcome this issue.

Cost is a very important factor when choosing the energy storage; there are also other factors to be taken into consideration, i.e. location for installation, efficiency rate and charging/discharging capacity. The location for the installation of BES is convenient, the choice of site is difficult for PHS. When there is a lot of energy needed to get charged within a short time period, CAES and PHS may not be the best choice as the charging capacity is very limited.

5.3 Carbon emission

In this section, I study the carbon emission modelling. It will include three subsections, a fixed carbon price, variable carbon prices and a constraint on carbon emission. The effect is illustrated using the Aberdeen application.

CO_2 is the main greenhouse gas and it is suspected to be the principle gas responsible for global warming and climate change. Different policies have been designed and implemented to incentivise the development of renewable energy sources with the goal of reducing CO_2 emissions. One commonly used method is carbon pricing, which is favoured by many economists for reducing global warming emissions. Pricing carbon emissions is also one of the incentives used by governments to encourage companies and households to produce less pollution by investing in cleaner technologies and adopting greener practices. It charges those who emit CO_2 for their emissions either in the form of a carbon tax or a cap and trade.

A carbon tax is defined as a fee placed on greenhouse gas emissions released from burning fuels. It can be done by setting a surcharge on carbon-based fuels and other sources of pollution, sending a price signal that will elicit a response over time which results in reduced emissions.

A cap and trade works in a way that governments put a limit (or cap) on the overall level of carbon pollution generated by industry and reduces that limit every year to reach a set pollution target. As the cap decreases each year, it then forces those who exceed their emission quota to buy from the others. The pollution quotas are created and distributed by the government through auctions and the price of quotas is determined by the market. Under this system, it creates an incentive for firms to reduce their emissions and sell the unused quotas.

To say which system is more effective, it depends on how each system is designed. For example, how strong is the economic incentive? To which emission sector does it apply? How is the revenue is used? Each system has its own advantages over the other; a cap and trade provides more certainty about the amount of emissions' reductions but little certainty about the price of emissions. A carbon tax provides greater certainty about the price but little certainty about the amount of emissions' reductions.

5.3.1 Fixed carbon price

Wind is carbon free so the only generator of concern is the gas-fired plant. The optimisation problem can now be formulated as: given a fixed carbon price, what would the optimal capacity be? For this point, I directly penalise in the objective function.

In my system, the carbon emission is already very low due to the restriction on the energy output from the gas-fired plant. To evaluate the cost of CO_2 mitigation, I keep track of the hourly CO_2 emissions of the gas generator. Emission from the gas-fired plant is calculated by applying an emission factor ζ to the quantity of electricity generated Y. For the gas-fired plant, the hourly CO_2 emissions is calculated as: $E_{co_2} = Y \times \zeta$.

The carbon emission cost is $C_{co_2} = P_{co_2} \times E_{co_2}$.

To reflect the importance of CO_2 emission, a penalty on CO_2 is added in the objective function (3.1), as shown below:

$$\sum_{t=1}^{n} \left(P_{co_2} \times E_{co_2} \right).$$

5.3.1.1 Numerical example

According to the Department of Energy and Climate Change, the current carbon price is £16 per tonne and the emission factor is 0.523 kgCO_2/kWh. The type of energy storage considered is battery storage.

Results and discussion

Let $(\overline{Y_r}^*, \overline{W_r}^*, \overline{S_r}^*)$ denote the vector of optimal solutions.

Type	Aberdeen
Gas-fired plant $\overline{Y_r}^*$	12
Wind turbine $\overline{W_r}^*$	15.24
Battery $\overline{S_r}^*$	30

Table 5.4: Optimal capacities for energy storage and generators

By re-running the algorithm for Aberdeen and the optimal capacities remain the same which is $(\overline{Y_r}^*, \overline{W_r}^*, \overline{S_r}^*) = (12, 15.24, 30)$. With an additional carbon penalty in the objective function, the total cost increased to £7.17 million. With the current carbon price, the optimal solution remains unchanged, it simply suggests to pay the carbon tax without altering the optimal capacity.

The contribution from the gas-fired plant is very limited to enable a greater supply from wind. I expect little or no change in the optimal solutions but a higher objective value. The current carbon price is too low to have an impact in a system like mine. The total cost has increased by £0.17 million for Aberdeen compared to the results in Section 4.3 where there is no penalty for carbon emission.

5.3.2 Variable carbon price

In this subsection, I apply a range of carbon prices and investigate the problem that, up to what carbon price, there will be a decrease in the gas-fired plant capacity? Theoretically, if the carbon price P_{co_2} increases, there would be a decrease in the gas-fired plant capacity, thus, a lower carbon emission. I solve this problem numerically by substituting different P_{co_2} and keeping the system feasible, in other words, it will always be able to generate a solution.

5.3.2.1 Numerical example

As shown in the previous numerical example, a carbon price of £16 per tonne is too low to make an impact. The investment cost of the gas-fired plant is much lower compare to the wind turbine and the battery storage. Hence, the carbon price would need to be high in order to see a decrease in the capacity of the gas-fired plant. I re-run the algorithm with increasing values of carbon price and find out at which price, I am able to see a reduction in the gas-fired plant. The effect is demonstrated using the Aberdeen example as I expect the behaviour in Rugby to be similar.

Results and discussion

Let $(\overline{Y_r}^*, \overline{W_r}^*, \overline{S_r}^*)$ denote the vector of optimal solutions.

The optimal solution remains the same until I increase the carbon price to £116, the optimal solution becomes $(\overline{Y_r}^*, \overline{W_r}^*, \overline{S_r}^*) = (11.9956, 15.32, 30)$ with a total cost of £8.27 million. At a carbon price of £116, there is a reduction of 0.004 in the gas-fired plant capacity. My system is designed to enable a high supply from the wind, so the carbon emission is already low. With a system that has low carbon emission, a much higher carbon price is required to see significant effect. For instance, I further increase the carbon price to £176, the optimal solution becomes $(\overline{Y_r}^*, \overline{W_r}^*, \overline{S_r}^*) = (11.96, 15.51, 30)$, there is a reduction of 0.04 in the gas-fired plant capacity.

In my system, the only reliable supply is from the gas-fired plant. Although the wind condition is good for Aberdeen, there are still periods with low wind speeds. For this reason, it favours the gas-fired plant as the primary concern is to ensure electricity demand is met at all times. Furthermore, the carbon tax is acting as a soft constraint which is not a condition that must be met. For the carbon tax to be very effective, the price will have to be high for it to choose wind or battery storage over the gas-fired plant.

5.3.3 Carbon emission constraint

In this subsection, I introduce a hard constraint on the carbon emission. I assume the carbon emission from the previous setting is 100% and is used as a baseline, now I wish to reduce 5% of the carbon emission from the baseline. The optimisation problem can now be formulated as: given a carbon emission limit, what would the optimal capacity be? For this point, I introduce the carbon constraint $\sum_{t=1}^{n} Y \times \zeta \leq 0.95E_0$, where E_0 is the carbon emission baseline.

5.3.3.1 Numerical example

The optimal capacity for the gas-fired plant from the original set-up for Aberdeen is 12 MWh and it delivers a constant output of 20% of its full generation capacity. So the annual emission $E_0 = 0.2 * 12 * 0.523 * 8736 = 10965$ tonnes. In order to not exceed the limit, the annual emission must be no more than 10417 (10965*0.95) tonnes. In other words, the electricity generated per time period should be no more than 2.28 MWh (10417/8736/0.523), and this implies that the capacity for the gas-fired plant should be no more than 11.4 MWh.

This carbon emission constraint puts a new upper limit on the gas-fired plant, which is lower than the previous upper limit. $Y_{\max}$ now equals to 11.4, it was 12 previously. With a decrease in the upper limit for the gas-fired plant, I expect an increase in the capacity of the wind turbine, or the battery storage, or both, depending on which solution generates the lowest possible total cost to cover the gap in the electricity generation.

Results and discussion

Let $(\overline{Y_r}^*, \overline{W_r}^*, \overline{S_r}^*)$ denote the vector of optimal solutions.

Type	Aberdeen
Gas-fired plant $\overline{Y_r}^*$	11.4
Wind turbine $\overline{W_r}^*$	30
Battery $\overline{S_r}^*$	30.83

Table 5.5: Optimal capacities for energy storage and generators

Re-run the model with the extra constraint on the carbon emission, it gives a solution of $(\overline{Y_r}^*, \overline{W_r}^*, \overline{S_r}^*) = (11.4, 30, 30.83)$ with a total cost of £8.27 million.

The carbon emission cap imposes a hard constraint and this forces the algorithm with no choice but to reduce the capacity of the gas-fired plant. With a hard constraint on the carbon emission constraint, the supply from the gas-fired plant is further limited. This means that it would need for a bigger wind turbine, a bigger battery, or both, to make up the difference. With a 5% carbon emission reduction from the baseline, the capacity for the gas-fired plant has decreased by 0.6 for Aberdeen.

In conclusion, in a system where there is limited input from a fossil fuel plant, a carbon tax needs to be very high to have a similar effect for the purpose of reducing emissions in comparison to a carbon emission cap.

5.4 Connection with the National Grid

In this section, I consider two extended models, one with the presence of the gas-fired plant (extended model a) and one without the gas-fired plant (extended model b). The carbon emission modelling from the previous section is not included as it creates lots of additional decisions and the effect of increasing carbon price may not be trivial.

5.4.1 Extended model a - with the gas-fired plant

I relax assumption (a) that was made at the beginning of Section 3.2 for my basic model by allowing import and export of electricity. That is, if the consumption exceeds the sum of the current electricity generation and the energy stored in the battery, then I can import the shortage from the National Grid; if there is a surplus and the storage device is full, I can export the surplus to the National Grid. The system is now represented in Figure 5.1.

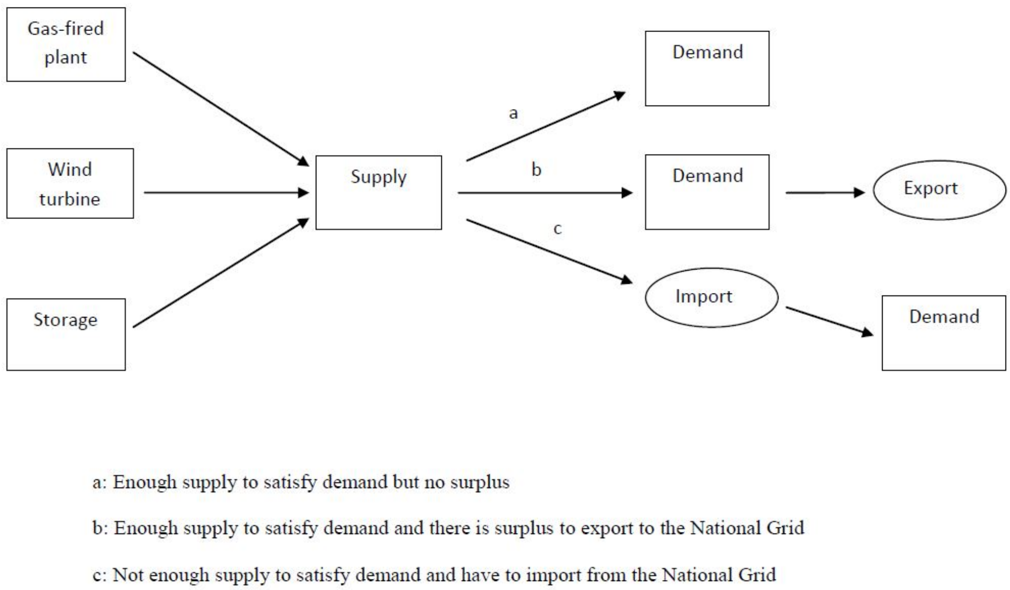

Figure 5.1: Diagram of the modelled system

I assume that a network connecting to the National Grid already exists, thus I do not look into the process of how to establish a network. I also do not consider payments from the National Grid for electricity generation capacity, I get paid only when I export to the National Grid. My system is mainly used to satisfy local electricity usage, not supply electricity to the National Grid. I make an additional assumption that electricity can be imported from or exported to the National Grid immediately. This is because when the battery storage is full and there is no additional storage to store the surplus, I would have to sell any surplus if I want to make some revenue. Similarly, if a deficit occurs during a particular period and the battery has been drained to its minimum level of charge, I need to import the additional unit from the National Grid immediately to fill up the shortage, as electricity demand must be met at every time period. Moreover, I set up the restriction that the National Grid makes up at most γ of the electricity consumption, thus, this forces my system to generate at least (1 - γ) of the electricity consumption, where $0 \leq \gamma < 1$. That is, if I have a deficit at time t, I deplete all the usable energy in the battery and then import the difference from the National Grid. So, $G_t = \min\big((D_t \times \Delta t) - W_t - Y - \rho_E S_{t-1}, \gamma(D_t \times \Delta t)\big)$.

On the other hand, if I have a surplus at time t, my first priority is to fill up the battery, I then export the remainder H_t to the National Grid. I introduce a degree of flexibility into the system by imposing the constraint that I export only a certain percentage ($0 \leq \beta < 1$) of the surplus. For instance, setting $\beta = 0.1$ amounts to exporting only 10% of the surplus. The export function $H_t = \beta(W_t + Y - D_t - S_t^+)$, where β is the percentage level of export.

The reason to set up a restriction of γ is that when importing from the National Grid, electricity procured from National Grid depends on how much electricity is generated from the gas-fired plant and the wind turbine, and on how much energy is stored in the battery. However, due to the nature of the optimisation objective, which is to minimise the total cost, the algorithm will always favour importing as much as possible from the National Grid because the electricity buying price is much less than the cost associated with the gas-fired plant, the wind turbine and the battery storage. Similarly, for β, if I export whatever the amount I have to the National Grid, the algorithm will suggest a maximum capacity for the gas-fired plant, the wind turbine and a minimum capacity for the battery. In this way, the battery can be filled up quickly, and the majority of the electricity demand can be satisfied by the gas-fired plant, so a maximal amount of surplus can be exported. However, this may cause congestion in the transmission line, and if all my export is accepted, the National Grid will have to pay somebody else to stop generating in order to maintain the demand-supply balance.

Meanwhile, I replace the last constraint in (3.3) with

$$D_t \times \Delta t \leq W_t + Y + S_t^- + G_t.$$

Connecting to the National Grid may incur other costs (positive or negative), such as costs for distribution or imports, revenue for selling surplus. Accordingly, I add the following term to the objective functional (3.1):

$$\sum_{t=1}^{n} \Big(P_e G_t - P_s H_t + (A_c + A_d H_t) \Big)$$

For each unit of electricity I import from the National Grid, I pay a price of P_e, thus, it generates a cost of $P_e G_t$ for each time period. Suppose the price of exporting each unit of electricity is P_s, I make a revenue of $P_s H_t$ for each period. A charge from distributors implies that I need to make a payment in order for the electricity to be distributed when exporting to the National Grid. This payment usually involves a fixed component and a component that varies with the quantity. Let me denote the charge to be $A_c + A_d H_t$, where A_c is the fixed component of the distribution charge and A_d is the variable component of the distribution charge.

5.4.2 Extended model b - without the gas-fired plant

After studying the scenario that the system is connected to the National Grid with the presence of the gas-fired plant, I now look at the scenario when the gas-fired plant is absent. The system diagram is shown in Figure 5.2.

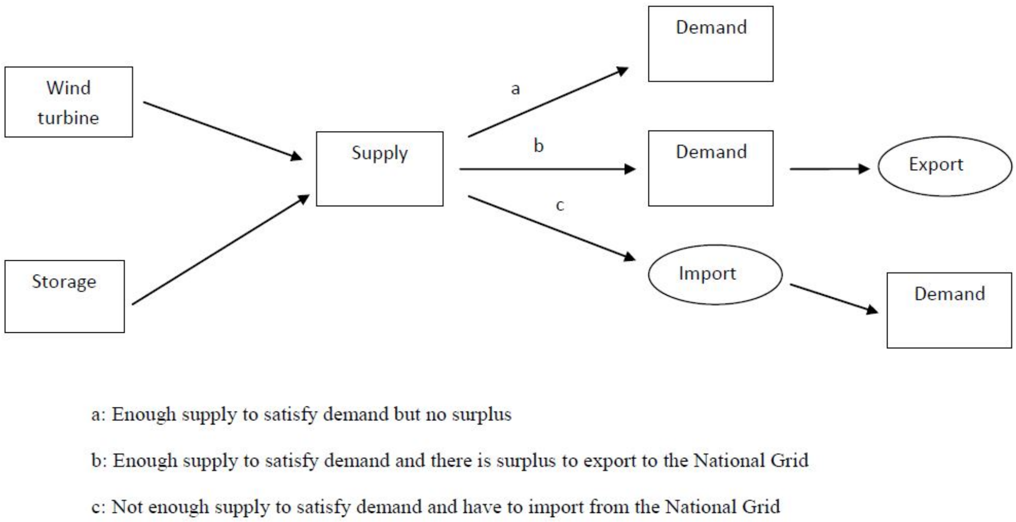

Figure 5.2: Diagram of the modelled system

As before, I assume a network connecting to the National Grid is already established, the main difference is that there is no input Y_t anymore. If I have a deficit at time t, $G_t = \min\big((D_t \times \Delta t) - W_t - \rho_E S_{t-1}, \gamma(D_t \times \Delta t)\big)$. On the other hand, if I have a surplus at time t and battery is full, the export function $H_t = \beta(W_t - D_t - S_t^+)$, where β is the percentage level of export.

I replace the last constraint in (3.3) with $D_t \times \Delta t \leq W_t + S_t^- + G_t$ and add the following term to the objective functional (3.1):

$$\sum_{t=1}^{n} \Big(P_e G_t - P_s H_t + (A_c + A_d H_t)\Big).$$

5.4.3 Numerical example

I provide numerical examples for the two scenarios, one with the gas-fired plant and one without the gas-fired plant. The other parameters are kept the same as the ones in Section 4.3. In this subsection, I will need to collect the following additional data.

1. Electricity buying and selling price

2. Electricity distribution price

The electricity buying and selling price, P_e and P_s (measured in £/MWh) are the system buying and selling price over the period from 1 January to 31 December 2012, downloaded from the Elexon website. Full information on this subject is contained in the section - settlement and trading charges of the balancing and settlement code (BSC). The price is collected as half hourly data and then averaged into an hourly price. The distribution cost differs from region to region. This was requested and obtained from the National Grid. The cost is made up of a fix component and a component that varies with quantity. For Aberdeen, the daily fixed component A_c is 5.45 p/kWh, whereas the variable cost A_d is 1.22 p/kWh during peak time, and 0.17 p/kWh during off-peak time. For Rugby, there is no fixed cost, $A_c = 0$. The variable cost A_d is 2.13 p/kWh during peak time and 0.43 p/kWh during off-peak time. All units are converted into £/MWh for consistency. In my study, I consider peak time as 7am - 7pm, and I set $\gamma = 0.1$, $\beta = 0.1$. The choice of energy storage is battery energy storage in these numerical examples.

5.4.3.1 Results and discussion for extended model a

I provide numerical examples for this extended model to make comparisons with the results obtained in Section 4.3. Let $(\overline{Y_r}^*, \overline{W_r}^*, \overline{S_r}^*)$ denote the vector of optimal solutions.

Type	Aberdeen	Rugby
Gas-fired plant $\overline{Y_r}^*$	12	11.85
Wind turbine $\overline{W_r}^*$	12.79	10
Battery $\overline{S_r}^*$	30	30

Table 5.6: Optimal capacities for energy storage and generators

With the above choice of parameter values, the optimisation algorithm returns an optimal capacity $(\overline{Y_r}^*, \overline{W_r}^*, \overline{S_r}^*) = (12, 12.79, 30)$ for Aberdeen with a total cost of £6.58 million. The optimal capacity is $(\overline{Y_r}^*, \overline{W_r}^*, \overline{S_r}^*) = (11.85, 10, 30)$ for Rugby with a total cost of £6.51 million over the year 2012.

By comparing to the results obtained in Section 4.3, the optimal solutions have changed. In Aberdeen, a smaller wind turbine can still guarantee the satisfaction of local electricity consumption with the help from imports for a few periods. For Rugby, with the help from imports, it releases some pressure from the gas-fired plant, but it cannot afford to reduce the wind turbine capacity. On the other hand, the surplus from wind which would otherwise be dumped can be sold to the National Grid to make some revenue after filling up the battery. Figure 5.3 shows the amount that was exported to the National Grid. Aberdeen is more consistent, exporting for almost 80% of the total time periods. Rugby exports for less than 50% of the total time periods with a relatively smaller amount. Based on the amount exported, the total cost has decreased by £0.42 million and £0.05 million for Aberdeen and Rugby, respectively. The system operates more efficiently as it makes better use of the surplus electricity.

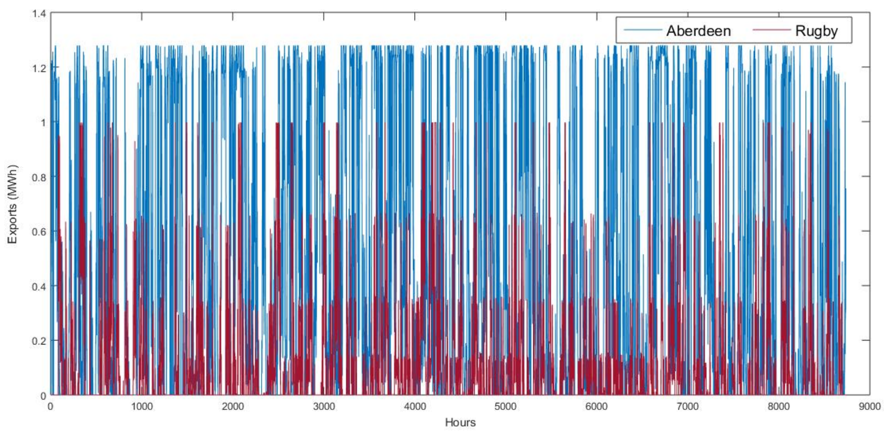

Figure 5.3: Amount exported to the National Grid from Aberdeen and Rugby

5.4.3.2 Results and discussion for extended model b

I now provide an example to compare with the results from the extended model a. Due to the absence of the gas-fired plant, I increase the import level to 50% of the consumption, so that my system will only have to supply 50% of the original electricity consumption. Other settings remain the same. Let $(\overline{W_r}^*, \overline{S_r}^*)$ denote the vector of optimal solutions.

Type	Aberdeen	Rugby
Wind turbine $\overline{W_r}^*$	91.24	359.44
Battery $\overline{S_r}^*$	112.60	121.51

Table 5.7: Optimal capacities for energy storage and generator

With the above choice of parameter values to supply 50% of the consumption, the optimisation algorithm returns an optimal capacity $(\overline{W_r}^*, \overline{S_r}^*) = (91.24, 112.60)$ and $(\overline{W_r}^*, \overline{S_r}^*) = (359.44, 121.51)$ for Aberdeen and Rugby, respectively. The total cost over the year 2012 is £23.07 million and £46.28 million for Aberdeen and Rugby, respectively. Without the constant supply from the gas-fired plant, it is not surprising to see a significant increase in the capacity of the wind turbine and the battery.

Chapter 6

Conclusion

In this study, I investigate a joint optimisation of generation and energy storage with the presence of wind. I formulate the problem into a cost minimisation framework. The objective is to determine optimal capacities for the generators and storage that minimise the associated total cost, providing electricity demand that is met at every hour. The nature of the problem is constrained non-linear optimisation and it is solved in MATLAB.

The thesis includes six topics, where topic 1 is developed by me, then extended to include three model extensions (topics 4 - 6), where topic 2 and 3 used existing methods.

1. Develop the basic optimisation model.
2. Model the stochasticity of wind speed using MC models.
3. Forecast electricity demand using a mathematical model.
4. Compare the total cost among the different types of storage technologies.
5. Model the carbon emission within the system.
6. Extend the system to include the connection with the National Grid.

The problem is first formulated as a deterministic model with known wind speed and electricity demand in topic 1, and then extend to include the stochastic modelling of wind speed and forecasting of electricity demand in topic 2 and 3. The uncertainty in the availability of wind is captured using a MC model due to its ability to capture time correlation and has been frequently used in the literature. Both first and second order MC are applied to 1-year hourly time series, and it shows that both models are able to reproduce the statistical characteristics satisfactorily. However, a second order MC produces relatively better results than a first order MC. To forecast the electricity demand, I use a three-step model proposed by (Filik et al., 2011) due to its unique ability in long-term forecasting with an hourly accuracy. In the first step, I model the yearly demand changes, then the modelling of weekly residual variations within a year, and followed by the hourly variations within a week. The final model is constructed by summing these three parts. The results indicate that this model successfully captures the patterns of electricity demand and produces a very satisfying output.

In topic 4, I study the characteristics of other storage technologies in addition to battery storage. A variety of storage technologies have been developed and I wish to investigate how the optimal solution would change when replacing the battery storage with other types of storages by modifying the corresponding cost functionals and efficiency rate. Re-running the algorithm, it can be seen that the optimal capacities have changed across the different storage technologies. Aberdeen has a more volatile wind speed, which can generate a lot of surplus electricity. With a lower charging capacity for CAES and PHS, it is suggested to have a bigger capacity in order to make better use of the surplus. Meanwhile, the cost for CAES and PHS is relatively lower, with sufficient surplus, it is more cost-effective to invest in the storage than in wind turbine and gas plant. On the other hand, flywheels are very expensive and the efficiency is slightly lower compared to batteries, therefore requiring a greater supply from the wind turbine. In Rugby, wind condition is more stable, the surplus generated is smaller in comparison to Aberdeen. After meeting its electricity consumption, there is little left to be charged. For the storage with a lower efficiency, it will just suggest a slightly bigger wind turbine to overcome this issue. Cost is a very important factor when choosing the energy storage, but there are other factors that should be taken into consideration. For example, if there is a lot of energy needed to get charged within a short period of time, CAES and PHS may not be the best choice as the charging capacity is very limited.

In topic 5, I take into account of the carbon emission by introducing a carbon tax and a carbon emission cap. I conclude that the current carbon price per tonne is too low to make an impact in a system with similar settings to mine. A much higher carbon price is needed in order to achieve a certain ambitious target. My system is designed to have a high penetration level of wind, the generation from the gas-fired plant is very limited, thus, the emission level from the system is already low. Applying a carbon price to achieve an even lower annual carbon emission level would require a very high carbon price to have an impact. In my system, a carbon price of £116 in the example for Aberdeen starts to see a reduction in the carbon emission level. A carbon emission cap acts as a hard constraint and it is able to force a reduction in the capacity for the gas-fired plant. For a carbon tax to have a similar effect for the purpose of reducing emissions in comparison to a carbon emission cap, it would need to be very high.

In topic 6, I increase the model flexibility by studying how the system could operate more efficiently and economically. I consider the possibility of connecting my system to the National Grid, where I import from the National Grid when my system suffers electricity shortages, and export the surplus to the National Grid providing that the electricity demand is met and the storage is fully filled. In this set-up, it shows that the system operates more efficiently and economically. Aberdeen can even afford to use a smaller wind turbine capacity with little help from the imports. The surplus from wind which would otherwise be dumped, can be sold to the National Grid to make some revenue after filling up the battery. I also look at the scenario without the presence of the gas-fired plant. Optimal capacities increase significantly with only 50% of the demand to satisfy.

The research limitations in my study are:

1. Due to the inaccessibility and unavailability of actual hourly electricity demand data at a regional level, the electricity demand data are scaled down from a national level. However, the actual regional hourly demand pattern may differ from the national demand pattern.

2. For the yearly modelling of electricity demand, I have the limitation that only a few years of data are available to test the model. From the eight data points, I can only deduce a decreasing trend. However, it is not clear whether it is a linear relationship. With a much longer period of data, I would be able to fit a model with more accuracy and predict more accurately.

3. The only reliable supply in my system is from the gas-fired plant, and its electricity generation is constrained to enable a greater contribution from the wind. This limits the model to generate a more realistic value for the carbon tax.

As for future works, a number of interesting topics are identified.

1. In the current set-up, only wind energy is considered for the renewable source, it is possible to introduce other types of renewable sources, i.e. solar and wave. In this way, there are more choices for the algorithm to allocate which leads to more interesting results.

2. Another interesting topic is to introduce a spot market, electricity is bought from and sold to the National Grid currently, but it is possible to adjust the framework to buy electricity from and sell to the spot market. Electricity auction would be a very interesting topic to study.

3. In this work, my gas-fired plant is set to deliver a constant output for every time period, regardless of the wind generation. It may be necessary to develop a more flexible function for the gas-fired plant generation, for instance, generating more when the wind contribution is low and vice versa.

4. The wind speed is modelled by a MC model, the dependence structure is reproduced. However, its ability to capture dependence structure decreases with increasing lags in time. This will be a very interesting topic for future research.

5. A MC model is also applied to the electricity demand in this work, but a standard MC is not effective in capturing the demand pattern (see Appendix A). Therefore, a more sophisticated MC model is required when modelling the electricity demand.

Appendix A

Electricity demand forecasting using Markov chain models

In this thesis, I have also considered using MC models to simulate the electricity demand. This technique has been used to model wind speed in Section 4.1, it is particularly suited to modelling systems where the current state of a sequence is highly correlated to the state immediately preceding it and where a large sample size of data exists. For both numerical applications, electricity demand is divided into 28 states based on the visual examination of the histogram of the demand data.

A.1 Electricity demand of Aberdeen

Descriptive statistics of the observed and generated demand are presented in Table A.1. Figure A.1 and A.3 represent the actual and synthetic data of the first order and second order MC models respectively. I also include the plots of the error terms of the two models (see Figure A.2 and A.4). The ACF plot of the observed and synthetic electricity demand data is in Figure A.5. The observed electricity demand time series shows a highly dependent structure from its oscillation behaviour and it has higher autocorrelation value at the same time lag than the synthetic electricity demand time series.

	Actual data	Synthetic data (1st order)	Synthetic data (2nd order)
Mean	51.67	51.19	51.33
Std deviation	10.87	10.57	11.22
Variance	118.18	111.66	126.07
Median	52.31	51.50	51.69
Minimum	26.46	28.02	26.02
Maximum	81.54	81.99	81.99

Table A.1: Descriptive statistics of synthetic and actual data

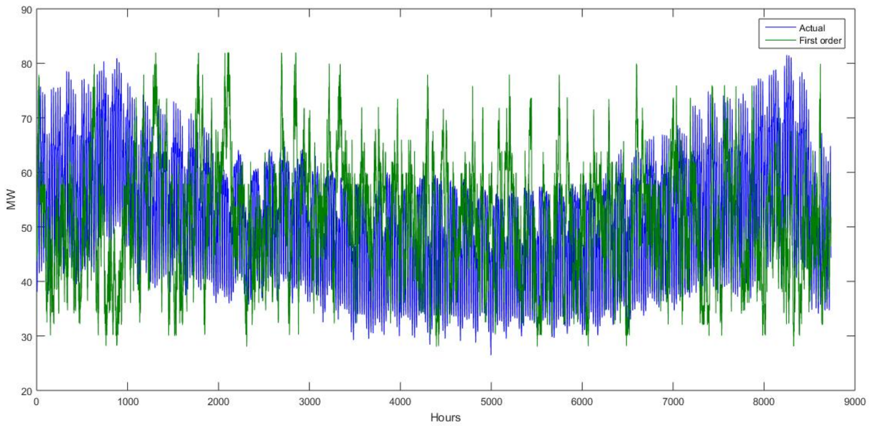

Figure A.1: Synthetic against actual electricity demand of 2012 - first order

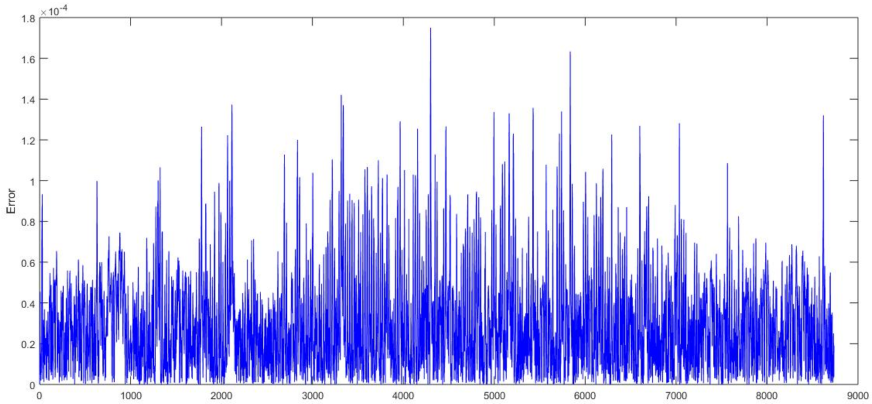

Figure A.2: Absolute value of error term for first order Markov chain model

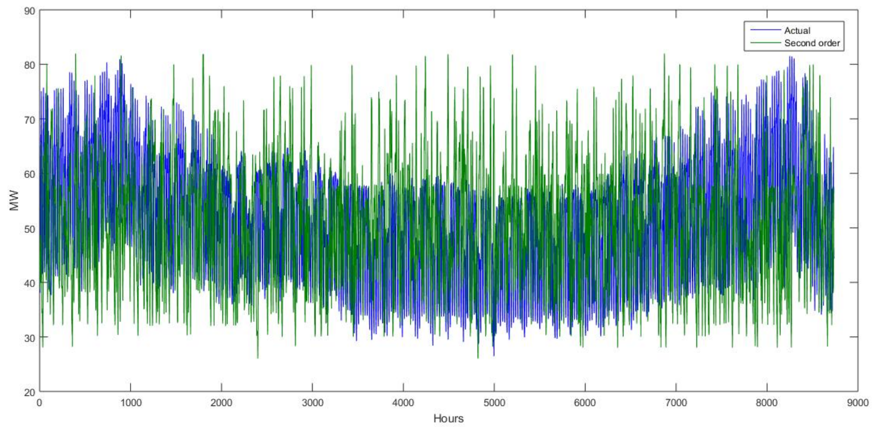

Figure A.3: Synthetic against actual electricity demand of 2012 - second order

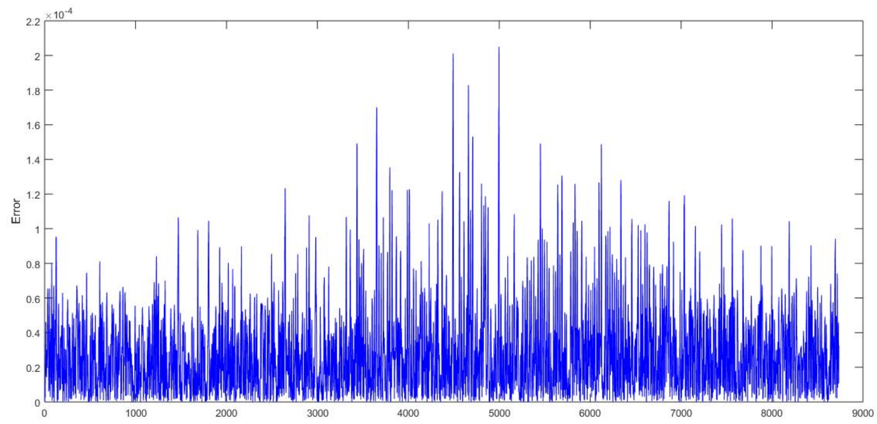

Figure A.4: Absolute value of error term for second order Markov chain model

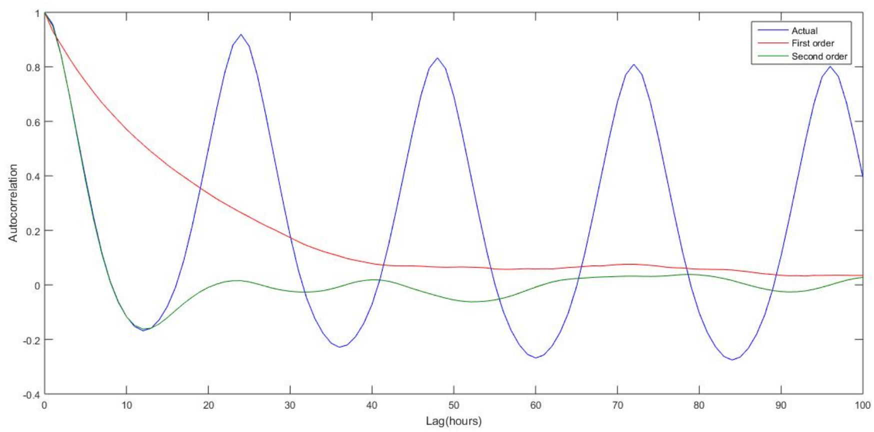

Figure A.5: Autocorrelation functions of observed and synthetic wind speed

A.2 Electricity demand of Rugby

Descriptive statistics of the observed and generated demand are presented in Table A.2, results generated by both MC models are similar. Figure A.6 and A.8 represent the actual and synthetic data of the first order and second order MC models respectively. And the error plots of the two models are shown in Figure A.7

and A.9. The ACF plot of the observed and synthetic electricity demand data (see Figure A.10) reports that the observed electricity demand time series has a highly dependent structure, and there is little correlation shown in the synthetic electricity demand time series.

	Actual data	Synthetic data (1st order)	Synthetic data (2nd order)
Mean	21.24	20.42	20.88
Std deviation	4.47	4.42	4.67
Variance	19.97	19.53	21.78
Median	21.51	20.44	21.21
Minimum	11.00	11.00	11.00
Maximum	33.52	33.96	33.79

Table A.2: Descriptive statistics of synthetic and actual data

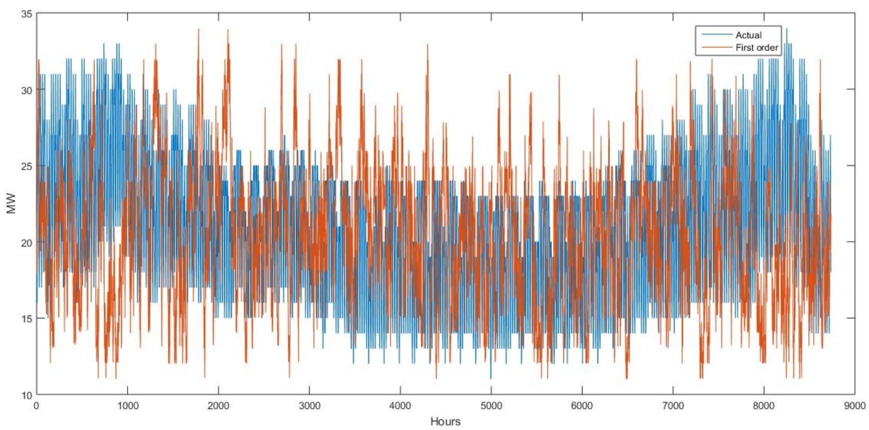

Figure A.6: Synthetic against actual electricity demand of 2012 - first order

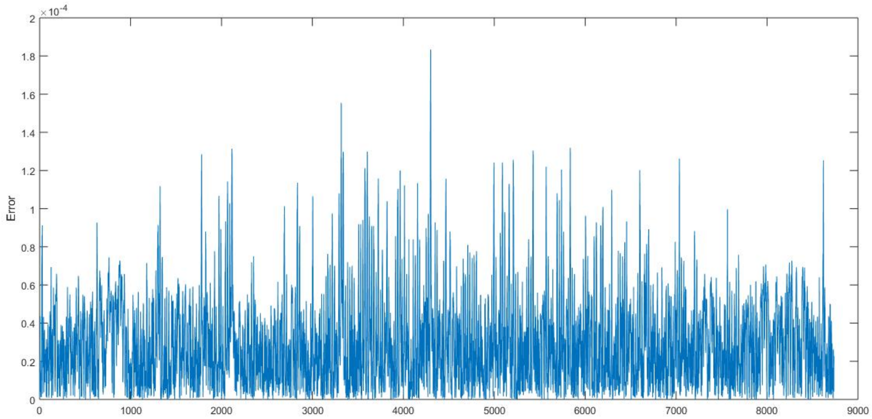

Figure A.7: Absolute value of error term for first order Markov chain model

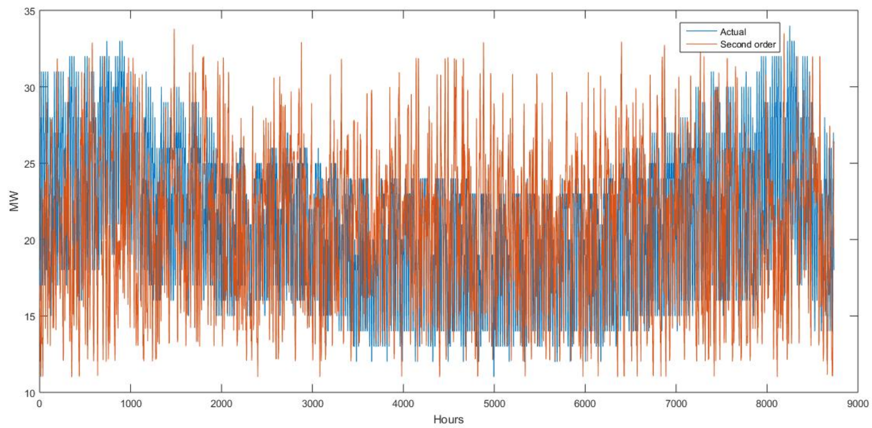

Figure A.8: Synthetic against actual electricity demand of 2012 - second order

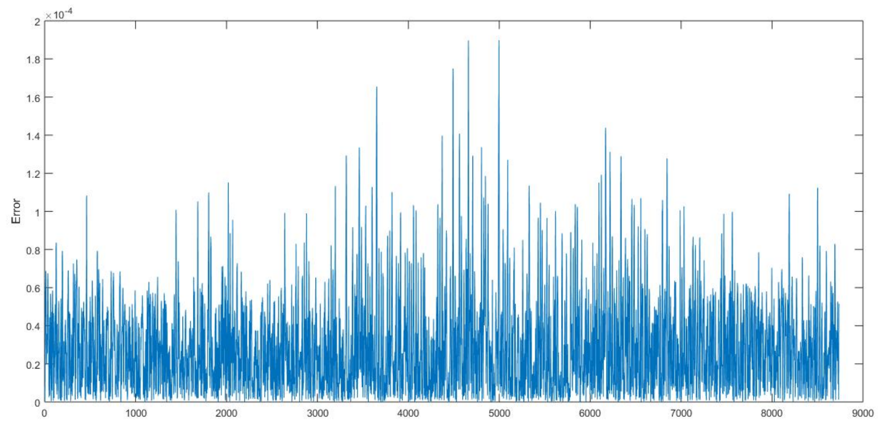

Figure A.9: Absolute value of error term for second order Markov chain model

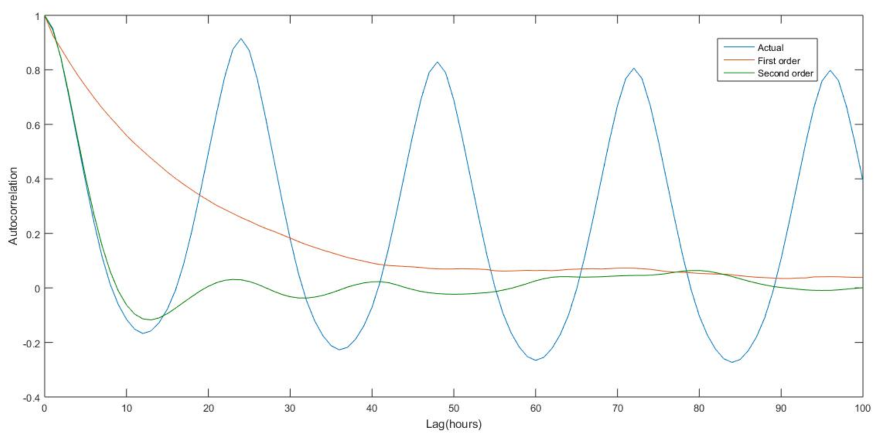

Figure A.10: Autocorrelation functions of observed and synthetic wind speed

Result and discussion

A MC model is used to model electricity demand for Aberdeen and Rugby using a 28 $\times$ 28 probability matrix. Both first and second order MC models have been applied to the available data. From the two numerical applications, I conclude that first and second order MC models do not produce promising results. Certain key statistical properties such as mean, standard deviation, maximum and minimum values are satisfactorily reproduced. However, the temporal properties of the synthetic sequence are poor compare with the observed data. In its current form of the MC model, it is unable to capture the variation at different times of the day. This is an obvious flaw to the model where the time of the day is a major determinant for electricity consumption. Therefore, daily peaks do not occur at the same time interval. The autocorrelation function is not reproduced and there is little correlation in the synthetic data. In order to produce more accurate results, a time component may have to be included as part of the transitional probability matrices.

Appendix B

Solutions for the optimisation problems

As there are quite a few optimisation problems studied in this thesis, leading to a number of numerical examples. For the readers' convenience, I collect and present the numerical solutions from each chapter in this appendix.

B.1 Numerical examples for Chapter 3

In Chapter 3, I have presented the optimisation model and provided two numerical examples for demonstration (one for Aberdeen and one for Rugby) in Section 3.3. The model inputs for the numerical applications in this chapter are: investment cost (C_{inv}) and O/M cost ($C_{\text{o/m}}$) measured in £/MWh for each plant; fuel price (P_g) for the gas-fired plant which measured in £/GJ; hourly electricity demand (D_t) measured in MW and the upper, lower generation capacity limits for each plant measured in MWh. I solve the optimisation problem in MATLAB. With $(\overline{Y_r}^*, \overline{W_r}^*, \overline{S_r}^*)$ denoting the vector of optimal solutions, the outputs for Aberdeen and Rugby are:

Type	Aberdeen	Rugby
Gas-fired plant $\overline{Y_r}^*$	12	12
Wind turbine $\overline{W_r}^*$	10.04	18.60
Battery $\overline{S_r}^*$	30	30.03
Total cost (M£)	6.57	7.28

Table B.1: Optimal solution for the deterministic model

B.2 Numerical examples for Chapter 4

In Chapter 4, I study the stochastic behaviour of wind speed and electricity demand forecasting. Rather than using real data, I simulate or predict the wind speed and electricity demand, while keeping the other parameters the same as in Chapter 3. The new optimal solutions are:

Type	Aberdeen	Rugby
Gas-fired plant $\overline{Y_r}^*$	12	11.93
Wind turbine $\overline{W_r}^*$	15.24	10
Battery $\overline{S_r}^*$	30	30
Total cost (M£)	7.00	6.56

Table B.2: Optimal solution for the stochastic model

B.3 Numerical examples for Chapter 5

In Chapter 5, I consider three extensions to increase the flexibility of my model and improve the efficiency of my system.

B.3.1 Different types of storage technology

First of all, I investigate how the optimal solution changes for different types of storages. Here, I input different round-trip efficiency, charging/discharging capacity, investment and O/M cost for each type of storage and compare the optimal solutions.

Type	BES	FES	PHS	CAES
Gas-fired plant $\overline{Y_r}^*$	12	12	12	11.84
Wind turbine $\overline{W_r}^*$	15.24	18.36	11.91	11.65
Storage $\overline{S_r}^*$	30	30	35	40
Total cost (M£)	7.00	15.05	2.41	2.23

Table B.3: Optimal solution under different storage technologies - Aberdeen

Type	BES	FES	PHS	CAES
Gas-fired plant $\overline{Y_r}^*$	11.93	12	12	12
Wind turbine $\overline{W_r}^*$	10	10.01	10.03	11.07
Storage $\overline{S_r}^*$	30	30	30	30
Total cost (M£)	6.56	14.37	2.20	2.16

Table B.4: Optimal solution under different storage technologies - Rugby

B.3.2 Carbon emission

Secondly, I include carbon emission modelling in the system by applying a carbon tax and emission cap. When implementing it, I add the following as model inputs, for example, the carbon price (£/tonne), the constraint on carbon emissions. The storage considered is battery storage. I used the Aberdeen example only to show how the concept works, the behaviour should be the same in Rugby.

By applying a fixed carbon tax for Aberdeen, it yields the same optimal solution as the one shown in Table B.2, but with a total cost of £7.17 million.

Then, I apply a range of carbon prices in increasing steps and see at which price it can cause a decrease in the gas-fired plant capacity. It turns out that at a price of £116, it starts to show a reduction.

Carbon price	116
Gas-fired plant $\overline{Y_r}^*$	11.9956
Wind turbine $\overline{W_r}^*$	15.32
Battery $\overline{S_r}^*$	30
Total cost (M£)	8.27

Table B.5: Optimal solution under the reference carbon price - Aberdeen

At last, I impose a constraint on carbon emission.

Type	Aberdeen
Gas-fired plant $\overline{Y_r}^*$	11.4
Wind turbine $\overline{W_r}^*$	30
Battery $\overline{S_r}^*$	30.83
Total cost (M£)	8.27

Table B.6: Optimal solution with carbon emission constraint - Aberdeen

B.3.3 The connection with the National Grid

Finally, I relax assumption (a) in Section 3.2 and look at the possibility of connecting the system to the National Grid, where I import from or export to the National Grid. In the meantime, I restrict the level of import and export. I consider two cases, one with the presence of the gas-fired plant (model a) and one without the gas-fired plant (model b). When implementing it in MATLAB, I further input electricity buying and selling price (P_e, P_s, measured in £/MWh), distribution cost A_c (daily), A_d, measured in £/MWh). In this section, carbon emission modelling is not considered as it will involve lots of decisions and the results might be very complicated to explain.

Example for model (a)

Type	Aberdeen	Rugby
Gas-fired plant $\overline{Y_r}^*$	12	11.85
Wind turbine $\overline{W_r}^*$	12.79	10
Battery $\overline{S_r}^*$	30	30
Total cost (M£)	6.58	6.51

Table B.7: Optimal solution for extended model with the presence of gas-fired plant

Example for model (b)

Due to the absence of the gas-fired plant, I increase the import level to 50% and other settings remain the same.

Type	Aberdeen	Rugby
Wind turbine $\overline{W_r}^*$	91.24	359.44
Battery $\overline{S_r}^*$	112.60	121.51
Total cost (M£)	23.07	46.28

Table B.8: Optimal solution for extended model without the presence of gas-fired plant

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